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INFORMATIONAL CASCADES ELICIT PRIVATE INFORMATION*

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We introduce cheap talk in a dynamic investment model with information externalities. We first show how social learning adversely affects the credibility of cheap talk messages. Next, we show how an informational cascade makes truth-telling incentive compatible. A separating equilibrium only exists for high-surplus projects. Both an investment subsidy and an investment tax can increase welfare. The more precise the sender's information, the higher her incentives to truthfully reveal her private information.

1. INTRODUCTION

A decision maker typically faces a lot of uncertainty when deciding over a course of action. For example, investors know they face the risk of losing all their money. Students do not know which University degree maximizes their future job market prospects. Consumers do not know which product offers the best price/quality ratio . . . To be more specific, suppose someone has the opportunity to invest in a project whose returns are positively correlated with the "general future health of the U.S. economy." Obviously, assessing the future state of the U.S. economy is a hard task and no human being is smart enough to make an errorless prediction about it. However, investors do not live like Robinson Crusoe—isolated on an island. Instead, they realize that the economy is populated by many other potential investors who all face the same type of risk. Moreover, they know that if they were to meet and exchange opinions, this would enable them to reduce their forecasting error. But if investors really care about one another's opinions, how will this information be disseminated throughout the economy?

Casual observation of everyday life suggests there are two different channels through which investors may learn about one another's opinions: One may learn through *words* or one may learn through *actions*. With the former, we have in

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mind a situation in which one investor simply tells her opinion to (possibly many) other investors. For example, every now and then managing directors of important companies appear in the media and express their opinions on a wide range of issues such as future technological developments, future oil prices, future market growth, etc. Some institutions are even specialized in collecting and summarizing the opinions of a large number of market participants. For example, the Munich-based IFO institute for economic research releases a quarterly index reflecting the business confidence of the average German investor. With learning through actions, we mean that if someone invests in a one-million-dollar project in the United States, this reveals her confidence in the American business climate.

In this article, we analyze the interaction between both communication channels. More specifically, we consider the following setup: N investors must take an investment decision and possess some private information concerning the future state of the economy. Investing is only profitable in the good state. For the sake of simplicity, we assume that the returns of the investment project only depend on the state of the economy. Hence, for reasons of efficiency all investors should truthfully exchange their private information. There are two investment periods. Prior to the first investment period, one investor appears in the media and is asked to divulge her private information to the other investors. Prior to the second investment period, everyone observes who invested in the first period. We compute all monotone stable perfect Bayesian equilibria[2] (MSPBE) of our game.

We first show that both communication channels do not coexist peacefully, in the sense that there does not exist an MSPBE in which the sender (i.e., the investor who appears in the media) truthfully announces her private information and in which subsequently a lot of information is generated through actions. This tension between both communication channels manifests itself differently depending on the surplus generated by the project: For low-surplus projects the unique MSPBE is the pooling one,[3] whereas for high-surplus projects there also exists an equilibrium in which the sender truthfully reveals her private information but in which "little" information is transmitted through actions.

The intuition behind this result goes as follows: In our model, expected payoffs are driven by the relative number of optimists in the economy (the higher the proportion of optimists in the population, the higher the probability that the world is in the good state). At time two, all players observe the number of period-one investments and use this knowledge to get an "idea" of the proportion of optimists in the economy. This updating process depends on the period-one investment strategies[4] (which are affected by the sender's message). If the investment only generates a low surplus, pessimists will—independently of the sender's message—never invest in the first period. Both sender's types then want to send the message

[2] Bluntly stated, in a monotone equilibrium we rule out the (unintuitive) possibility that pessimistic investors are more likely to invest (in the first investment period) than optimistic ones.

[3] In this equilibrium, no credible information is transmitted through words, but "a lot" of information is transmitted through actions.

[4] For example, on observing k period-one investments, players compute different posteriors if pessimists invested (at time one) with zero probability and optimists with a probability equal to one, than if pessimists invested with the same probability as the optimists.

that makes the optimists invest with as large a probability as possible.[5] Thus, both sender's types share the same preferences over the receivers' actions, and therefore no information can be transmitted through cheap talk. For high-surplus projects, however, this intuition is incomplete. In that case, all players face a positive gain of investing after receiving the message "I am an optimist." If a player then believes that everyone will invest at time one, it's optimal for her to do so too (i.e., an informational cascade[6] in which everyone invests is ignited by the arrival of a favorable message). In our model, this informational cascade induces a pessimist to send the message "I am a pessimist": If she were to deviate and sent instead the message "I am an optimist," she would not be able to learn anything about the proportion of optimists in the population and would never invest. An optimist faces a high opportunity cost of waiting and, independently of her message, invests at time one. Hence, she cannot gain by sending the message "I am a pessimist."[7]

Our analysis allows us to draw some positive and normative conclusions. In particular, we show that an investment subsidy, by artificially increasing the surplus generated by the project, promotes truthful revelation of private information. However, this does not mean that an investment subsidy always increases welfare: A social planner knows that if the subsidy induces truthful revelation, this comes at the cost of less information transmission through actions. In the article, we show that a social planner may even want to tax investments to cause information to be revealed through actions instead of words. Finally, we also show that a more able sender (i.e., a sender possessing a more precise signal) has more incentives to truthfully reveal her private information than a less able one.

This article belongs to the literature on informational cascades (see among others Banerjee, 1992; Bikhchandani et al., 1992 (BHW hereafter); Chamley and Gale, 1994; Chamley, 2004a; for an excellent overview and introduction to this literature see Chamley, 2004b). Those papers assume away any preplay communication and study the efficiency properties of social learning (learning takes place through actions only). Our results provide a justification for this approach: For low-surplus projects, no information can be transmitted through words because players want to influence their future learning capabilities. In those papers, the public information is the consequence of some costly actions undertaken by the early movers: For example, a second mover knows that the first mover is an optimist because she spent money to undertake a new investment project. Hence, in those papers the credibility of the public information is not an issue. In this article it is costless to send public information, and its credibility must therefore be carefully checked. Those papers show how an informational cascade develops

[5] If the sender succeeds for example in making the optimistic receivers invest with probability one, she perfectly learns the proportion of optimists in the population.

[6] All players—irrespective of their private information—rely on the public information (i.e., the message of the sender) and take the same action at time one. By definition, this is an informational cascade.

[7] Note that in the separating equilibrium, information only gets transmitted through actions when the sender announces "I am a pessimist." As will become clear below, the amount of information produced after the arrival of an unfavorable message is always lower than the one that would have been produced in the absence of cheap talk (or in the pooling equilibrium).

as a consequence of the arrival of some early (and credible) information. In this article, we show that the causality can also be reversed: It is the informational cascade, by reducing the gain of sending the message "I am an optimist," which causes the public information to be credible.

Doyle (2002) also introduces a social planner in a dynamic investment model with information externalities but without cheap talk. In contrast to our paper, pessimistic players do not possess an investment option and therefore never invest. Hence, Doyle's model does not feature an equilibrium in which pessimistic players invest at time one and consequently blur the information contained in all players' time-one investment decisions. Therefore, in his model, one would never want to tax investments.

Gill and Sgroi (2003) analyze a setup in which a possibly "optimistic," "pessimistic," or "unbiased" sender is asked whether or not to endorse a product. On hearing the sender's message, receivers decide sequentially whether or not to buy the product. Hence, in their model, receivers also learn through other receivers' actions and through the sender's message. In contrast to our article, the authors assume that the sender does not want to learn about the receivers' types (because, for instance, she already consumed the product and received her payoff). Therefore, she cannot gain by misrepresenting her private information.[8]

Obviously, this is not the first paper to investigate the credibility of cheap talk statements. In a seminal paper, Crawford and Sobel (1982) already analyzed the issue of information transmission through cheap talk. However, in their model, the receiver chooses an action that influences both players' payoffs after having received a message from the informed sender. In our model, the sender first sends a message and then plays a (waiting) game with the receivers. Farrell (1987, 1988), Farrell and Gibbons (1989), and Baliga and Morris (2002) also assume that both players play a game after having received or sent a message. However, they consider a very different game: in Farrell (1987, 1988) and Baliga and Morris (2002), the communication stage is followed by a coordination game, whereas in Farrell and Gibbons (1989), both players engage in a bargaining game after the communication stage. As we consider a (very) different game, we also get very different results: Crawford and Sobel (1982) have shown how the credibility of cheap talk statements are undermined when the sender and the receiver have different preferences over the optimal action, Baliga and Morris (2002) argued that positive spillovers impede information exchange, whereas we show how social learning may destroy incentives for truth-telling (and how informational cascades help in restoring these incentives).

This article is organized as follows. In Section 2, we present our two-stage game. In Section 3, we take the players' posteriors as given and solve for all monotone stable continuation equilibria. The proofs of the results stated in this section tend to be quite lengthy and we therefore decided not to include them in this article. We refer the interested reader to Gossner and Melissas (2003). We next compute equilibrium strategies in the sender–receiver game (Section 4). We first show

[8] Sgroi (2002) analyzes a similar setup and computes the optimal number of senders. As in Gill and Sgroi (2003), the senders are not interested in the receivers' signals.

how the credibility of cheap talk may be undermined when players can postpone their investment decisions (Proposition 4). Next, we show how this credibility can be restored by an informational cascade (Proposition 5). In Section 5, we discuss some normative and positive implications of our theory. Final comments are summarized in the sixth and final section.

2. THE MODEL

Assume that a population of $N \geq 5$ risk neutral players must decide whether to invest in a risky project or not. $V \in \{1, 0\}$ denotes the value of the investment project. The state of the economy is described by $\Theta \in \{G, B\}$. If $\Theta = G$ the good state prevails and $V = 1$, whereas if $\Theta = B$, the economy is in a bad state and $V = 0$. The prior probability that $\Theta = G$ equals 1/2. The cost of the investment project is denoted by c. Each player receives a private, conditionally independent signal concerning the realized state of the world. Formally, player l's signal $s_l \in \{g, b\}$ $(l = 1, \dots, N)$, where $\Pr(g \mid G) = \Pr(b \mid B) = p > 1/2$. We assume that

$$\text{A1}: \quad 1 - p < c < p.$$

A1 implies that a player who received signal g is, a priori, willing to invest ($\Pr(G \mid g) = p > c$), and that a player who received a signal b is, a priori, not willing to invest ($\Pr(G \mid b) = 1 - p < c$). Henceforth, we call a player who received a good (bad) signal an optimist (pessimist).[9] If $c \leq 1/2$ ($c > 1/2$), we call the investment opportunity a high (low) surplus project. We analyze the stage game that unfolds as follows:

−1. The state of nature is realized and players receive signals.
0. A randomly selected player i is asked to report her signal. Her message, $\hat{s}_i \in \{g, b\}$, is made public to all the other players.
1. All players make investment decisions.
2. All players observe who invested at time one, and those who haven't invested yet make new investment decisions.
3. All players learn the true state of the world. Payoffs are received and the game ends.

In the first stage (time zero), player i (the sender) influences the time-one posteriors of the remaining players (the receivers). Henceforth, we call the second stage the waiting game (or the continuation game). At time one, player l must choose an action, a_l, from the set {invest, wait}. At time two, all players who waited at time one must choose an action from the set {invest, not invest}. Each player only possesses one investment opportunity, so a period-one investor cannot invest in a second project at time two. Investments are irreversible. If a player does not invest

[9] Observe that in our model, all players are Bayesian rational: Optimists (pessimists) do not overestimate (underestimate) the probability that $\Theta = G$. Hence, our definitions differ from the ones that are used by behavioral economists. However, these definitions are intuitive and should not confuse the reader.

in any of the two periods, she gets zero. Investment decisions at period one are represented by a N-vector x, where the lth coordinate equals 1 if player l invested at time one and zero otherwise. δ denotes the discount factor.

We let h_t $(t = 0, 1, 2)$ denote the history of the game at time t. Thus, $h_0 = \{\emptyset\}$, $h_1 = \hat{s}_i$, and $h_2 = (\hat{s}_i, x)$. H_t denotes the set of all possible histories at time t, and the set of histories is $H = \bigcup_{t=0}^{2} H_t$. A symmetric behavioral strategy for the receivers is a function $\rho : \{g, b\} \times H \to [0, 1]$ with the interpretation that $\rho(s_j, h_t)$ represents the probability of investing at date t given s_j and h_t ($j = 1, \ldots, N$ and $j \neq i$). For instance, $\rho(g, b)$ is the probability that an optimistic receiver invests at time one given that $\hat{s}_i = b$, and $\rho(b, g)$ is the probability that a pessimistic receiver invests at time one given that $\hat{s}_i = g$. Since each player can only invest once, $\rho(s_j, h_2) = 0$ if player j invested at time one, and $\rho(s_j, h_0) = 0$ since no one can invest at time zero. A behavioral strategy for the sender is a function $\sigma : \{g, b\} \times H \to [0, 1]$. $\sigma(g, h_0)$ ($\sigma(b, h_0)$) represents the probability with which an optimistic (pessimistic) sender sends $\hat{s}_i = g$. $\sigma(\cdot, h_1)$ ($\sigma(\cdot, h_2)$) represents the probability that player i invests at date one (two). As before, $\sigma(\cdot, h_2) = 0$ if the sender invested in the first period.

When solving our game, we rely on four equilibrium selection criteria. First, we require a candidate equilibrium to belong to the class of the perfect Bayesian equilibria. Henceforth, $\sigma^*(\cdot)$ ($\rho^*(\cdot)$) denotes the value taken by $\sigma(\cdot)$ ($\rho(\cdot)$) in a perfect Bayesian equilibrium (PBE). In a PBE, strategies and beliefs (concerning the other players' types) must be such that (i) the sender cannot gain by choosing a $\sigma \neq \sigma^*$ given her beliefs and given ρ^*, (ii) receivers cannot gain by choosing a $\rho \neq \rho^*$ given their beliefs and given σ^*, and (iii) beliefs must be computed using Bayes's rule whenever possible. As usual, a pooling equilibrium is a PBE in which $\sigma^*(g, h_0) = \sigma^*(b, h_0)$. In that case, the message $\hat{s}_i = g$ is as likely to come from an optimistic as from a pessimistic sender. Hence, in that case, messages have no informational content and do not affect posteriors. For the sake of concreteness (and without loss of generality), we assume that $\sigma^*(g, h_0) \geq \sigma^*(b, h_0)$. This assumption merely defines message $\hat{s}_i = g$ as the one that influences posteriors in a (weakly) favorable way. Under this assumption, a separating equilibrium is a PBE in which $\sigma^*(g, h_0) = 1$ and $\sigma^*(b, h_0) = 0$. Note that at time one the posterior of the receivers may differ from the sender's. Therefore, we do not impose $\sigma^*(g, h_1)$ to be equal to $\rho^*(g, h_1)$. Similarly, we allow $\sigma^*(b, h_1)$ to be different from $\rho^*(b, h_1)$.

Second, we restrict ourselves to the class of the *monotone* strategies. Consider players l and l' (where l or l' may be the sender). Let $q \equiv \Pr(G \mid s_l, \hat{s}_i)$ (respectively, $q' \equiv \Pr(G \mid s_{l'}, \hat{s}_i)$) denote player l's (respectively, player l''s) time-one posterior. Strategies are said to be monotone if they possess the following two properties: (i) if $q = q'$, then Pr(l invests at time one) = Pr(l' invests at time one), (ii) if Pr(l invests at time one) > Pr(l' invests at time one), then $q > q'$. Remark that from the first property, monotone strategies are symmetric. Note also that the first property implies that whenever the sender's message is uninformative, the sender invests at time one with the same probability as a receiver of the same type, which need not hold in symmetric strategies. Property two implies that the time-one investment probabilities (weakly) increase in the time-one posteriors. Below, we will explain in more detail our need to focus on monotone strategies.

Third, we discard unstable equilibria. With "unstable," we refer to the traditional notion according to which an equilibrium is unstable if a small change in the investment probability of the other players induces a change in player l's optimal investment probability with the same sign and with a greater magnitude. This equilibrium selection criterion has also been used in the study of coordination problems (see, e.g., Cooper and John, 1988; Chamley, 2003). Chamley (2004a) already noted their existence in games with social learning. This requirement will also be explained in more detail below.

Finally, we require every candidate equilibrium to be robust to the introduction of an ϵ-reputational cost. More specifically, we assume that with probability ϵ_1, receivers detect any "lie" (i.e., the optimistic sender who sends message $\hat{s}_i = b$, or the pessimistic sender who sends message $\hat{s}_i = g$) from the sender, in which case she suffers a reputational cost equal to ϵ_2. It is important to note that ϵ_1 is unrelated to the sender's behavior in the continuation game. This assumption ensures that the sender's behavior in the continuation game is only driven by informational reasons (and not by her desire to "mask" a past lie). Let $\epsilon \equiv \epsilon_1.\epsilon_2$ and we assume that ϵ represents an arbitrary small, but strictly positive, number. With this reputational cost, an optimistic sender prefers to send a favorable to an unfavorable message (as will become clear below, in the absence of this ϵ, she would be indifferent between the two messages).

An MSPBE is a tuple of strategies and beliefs that satisfy our four equilibrium selection criteria.

3. STRATEGIC WAITING

Before proving the existence of a PBE in our game, we analyze equilibrium behavior in the continuation game. We restrict ourselves to the class of the *monotone stable continuation equilibria* (MSCE). Henceforth, $\tilde{\sigma}(\cdot)$ ($\tilde{\rho}(\cdot)$) denotes the value taken by $\sigma(\cdot)$ ($\rho(\cdot)$) in an MSCE. An MSCE is identical to an MSPBE except that we do not require the sender to choose $\tilde{\sigma}(g, h_0)$ and $\tilde{\sigma}(b, h_0)$ optimally given her beliefs and given equilibrium behavior in the continuation game. Stated differently, in an MSCE, we do not endogenize the receivers' time-one posteriors. Instead, we just treat them as if they were exogenous and analyze equilibrium behavior in the continuation game given players' posteriors. Note that every MSPBE is an MSCE, whereas the contrary need not hold.

In our companion paper (Gossner and Melissas, 2003), we characterized the set of MSCEs for all possible time-one posteriors. Unfortunately, this characterization required a quite lengthy, technical, and not very interesting exposition. To avoid this lengthy exposition, in this article we "only" intuitively explain our most important results. Moreover, when providing an intuition, we often restrict our attention to the limit case in which (i) the sender is an optimist who truthfully reports her private information and (ii) receivers compute their posteriors under the assumption of truthful revelation. In this limit case, optimistic receivers possess two favorable pieces of information and compute

$\Pr(G \mid s_j = g, \hat{s}_i = g) = p^2/(p^2 + (1-p)^2) \equiv \bar{q}$. Pessimistic receivers possess two contradictory pieces of information and compute $\Pr(G \mid s_j = b, \hat{s}_i = g) = 1/2$.

Our model is void of any competition effects or positive network externalities. Hence, a player's expected gain of investing is solely determined by the relative number of optimists (as compared with the number of pessimists) in the population. Denote by n the random number of optimists in our population. The higher n, the higher $\Pr(G \mid n)$ and the higher the expected gain of investing. Unfortunately, by postponing one's investment decision, players observe x, the vector of time-one investment decisions, instead of n. Hence, at time two, all players who waited at time one face an inference problem: On the basis of x, they must try to get "as precise an idea" about n.

As we only consider symmetric strategies, player i does not care about *who* invests, but rather in *how many* players invest. Therefore, from the sender's point of view, all information contained in x can be summarized by k^s (the number of receivers who invest at time one).[10] Similarly, from a receiver's point of view, all information contained in x can be summarized by k (the number of remaining receivers who invest at time one) and a_i (the time-one action of the sender).

We thus continue our analysis by working with k, k^s, and a_i. If player j waits, she observes k and a_i and invests if $\Pr(G \mid q, k, a_i) \geq c$. Hence, for a given k and a_i, player j's payoff equals $\max\{0, \Pr(G \mid q, k, a_i) - c\}$. Of course, player j cannot ex ante know the realization of k and a_i. Therefore, player j's ex ante gain of waiting (net of discounting costs), $W(q, \sigma_1, \rho_1)$, equals

$$W(q, \sigma_1, \rho_1) = \sum_{a_i} \sum_{k} \max\{0, \Pr(G \mid q, k, a_i) - c\} \Pr(k \mid q, a_i) \Pr(a_i \mid q) \tag{1}$$

where $\rho_1 \equiv (\rho(b, h_1), \rho(g, h_1))$ and $\sigma_1 \equiv (\sigma(b, h_1), \sigma(g, h_1))$. Similarly, player i's gain of waiting, $W(q, \rho_1)$, equals

$$W(q, \rho_1) = \sum_{k^s} \max\{0, \Pr(G \mid q, k^s) - c\} \Pr(k^s \mid q) \tag{2}$$

To gain some insight behind Equations (1) and (2), it is useful to consider Equation (1) when $q = \bar{q}$ (i.e., when player j is an optimist who believes the sender to be optimistic as well), $\sigma_1 = (0, 0)$ (i.e., when the sender invests with probability zero), and $\rho_1 = (0, \rho(g, g))$ (i.e., pessimistic receivers wait, whereas the optimistic ones invest with probability $\rho(\cdot)$). Equation (1) can then be rewritten as

$$W(\bar{q}, (0,0), (0, \rho(g, g))) = \sum_{k} \max\{0, \Pr(G \mid \bar{q}, k, \text{wait}) - c\} \Pr(k \mid \bar{q}, \text{wait}) \tag{3}$$

Suppose that $\rho(g, g) = 0$. If player j waits, she will then observe zero investments and compute $\Pr(G \mid \bar{q}, 0, \text{wait}) = \bar{q}$. This is intuitive: Player j, independently of n, always observes zero period-one investments. Stated differently, if $\rho\,(g, g) = 0$, it is

[10] In mathematical terms, we mean that $\Pr(n \mid x, s_i) = \Pr(n \mid k^s, s_i)$, $\forall n$.

as if she does not receive any additional information concerning the realized state of the world. Therefore, she has no reason to change her posterior and $\Pr(G \mid \bar{q}, 0, \text{wait}) = \bar{q}$. Hence,

$$W(\bar{q}, (0,0), (0,0)) = \bar{q} - c$$

Suppose now that $\rho(g, g) = 1$. Then, in the next period, player j learns how many optimists are present in the economy (i.e., $n = k + 2$).[11] At time two, player j computes $\Pr(G \mid n)$, and invests if $\Pr(G \mid n) \geq c$. As before, player j cannot ex ante know how many optimists are present in the economy, and therefore

$$W(\bar{q}, (0,0), (0,1)) = \sum_n \max\{0, \Pr(G \mid n) - c\} \Pr(n \mid \bar{q}) \tag{4}$$

LEMMA 1. $\forall\, \sigma_1, W(q, \sigma_1, (0, 1)) > q - c$.

PROOF. See Gossner and Melissas (2003). To gain some intuition behind Lemma 1, we explain why $\forall c \in (1-p, p)$, $W(\bar{q}, (0,0), (0,1)) > \bar{q} - c$ whenever our economy consists of at least five players. We can rewrite player j's gain of investing as follows:

$$\bar{q} - c = \sum_n \Pr(G \mid n) \Pr(n \mid \bar{q}) - c$$

Suppose $\rho_1 = (0, 1)$ and assume that player j decides to wait at time one and then to invest unconditionally (i.e., to invest at time two independently of n). The above equality merely states that investing at time one is payoff-equivalent (net of discounting costs) to *unconditionally* investing at time two. Equation (4) teaches us that waiting (when $\rho_1 = (0, 1)$) is equivalent to making an optimal *conditional* second-period investment decision. Observe that n cannot take a value lower than two because both players j and i are assumed to be optimists. If $\Pr(G \mid n = 2)$ is higher than or equal to c, then the optimal conditional second-period investment decision always coincides with unconditionally investing at time two. This means that $\bar{q} - c$ is equal to $W(\bar{q}, (0,0), (0,1))$. Hence, $W(\bar{q}, (0,0), (0,1))$ is strictly greater than $\bar{q} - c$ if (and only if) $\Pr(G \mid n = 2) < c$. In this model, all players possess a signal of the same precision. Therefore, $\forall\ c \in (1-p, p)$, it takes three pessimistic receivers to restrain an optimist, who learned through the sender's message that $s_i = g$, from investing (and therefore N must be greater or equal than five).

To focus on the interesting parameter range, we assume:

$$\text{A2}: \quad \frac{\bar{q} - c}{W(\bar{q}, (0,0), (0,1))} < \delta < 1.$$

[11] By assumption, player j is an optimist who waited at time one. Moreover, we analyze a case in which player j learned (through the sender's message) that $s_i = g$. Therefore, $n = k + 2$.

The first inequality of A2 puts a lower bound on the discount factor δ such that an optimistic receiver, who learned (through the sender's message) that $s_i = g$, faces a positive option value of waiting (i.e., if player j expects all the optimistic receivers to invest and all the other players to wait, then she rather waits). The first inequality ensures thus that $\tilde{\rho}(g, g) < 1$. The second inequality ensures that $\tilde{\rho}(g, g) > 0$.

LEMMA 2. $\forall \rho'(g, h_1) > \rho(g, h_1)$, $W(q, \sigma_1, (0, \rho'(g, h_1))) \geq W(q, \sigma_1, (0, \rho(g, h_1)))$, *and there exists a value* $\rho^c(q)$ *such that the inequality becomes strict whenever* $\rho'(g, h_1) > \rho^c(q)$ $(\rho^c(q) \in [0, 1))$.

PROOF. See Gossner and Melissas (2003). From Lemma 2 follows:

COROLLARY 1. $\forall\ \rho'(g, h_1) > \rho(g, h_1)$,

(i) $W(p, (0, \rho'(g, h_1))) \geq W(p, (0, \rho(g, h_1)))$, *where the inequality becomes strict whenever* $\rho'(g, h_1) > \rho^c(p)$ $(\rho^c(p) \in (0, 1))$,
(ii) $W(1 - p, (0, \rho'(g, h_1))) > W(1 - p, (0, \rho(g, h_1)))$.

PROOF. See Gossner and Melissas (2003). A slightly different version of Corollary 1 was already proven in Chamley and Gale (1994, Proposition 2). To understand the intuition behind Lemma 2 and Corollary 1, compare the following two "scenarios." In scenario one, all optimistic receivers randomize with probability $\rho'(g, g)$, and in scenario two, all optimistic receivers randomize with probability $\rho(g, g) < \rho'(g, g)$. Denote by n^r the number of optimistic receivers. Call k' (respectively, k) the number of players investing at time one when $n^r - 1$ optimistic receivers invest with probability $\rho'(g, g)$ (respectively, $\rho(g, g)$). Now, having $n^r - 1$ players investing with probability, $\rho(g, g)$ is ex ante equivalent to the following two-stage experiment: First, let all $n^r - 1$ players invest with probability $\rho'(g, g)$; next let all k' investors rerandomize with probability $\frac{\rho(g,g)}{\rho'(g,g)}$. Therefore, the statistic k is generated by adding noise to the statistic k'. Therefore, k' is a sufficient statistic for k. From Blackwell's value of information theorem (1951), we know that this implies that $W(\bar{q}, (0, 0), (0, \rho'(g, g))) \geq W(\bar{q}, (0, 0), (0, \rho(g, g)))$. Lemma 2 states that the inequality becomes strict once $\rho'(g, g)$ passes a critical threshold level.

Stated differently, $\rho(g, g)$ captures the ex ante amount of information produced by the optimistic receivers. The higher $\rho(g, g)$, the easier one can infer n out of k (this can best be seen by comparing the two polar cases where $\rho(g, g) = 0$ and $\rho(g, g) = 1$; see above) and thus the higher the ex ante gain of waiting.

PROPOSITION 1. *If the investment generates a low surplus and if* $\Pr(G \mid s_j = g, \hat{s}_i = g) > p$, *there exists a unique MSCE in which the sender and the pessimistic receivers wait, whereas the optimistic receivers invest with probability* $\tilde{\rho}(g, g) \in (0, 1)$.

PROOF. See Gossner and Melissas (2003). To understand the intuition behind Proposition 1, we focus on our limit case in which $\Pr(G \mid g, g) = \bar{q}$. As $c > 1/2 = \Pr(G \mid b, \hat{s}_i = g)$, no pessimist wants to invest at time one. Suppose the optimistic

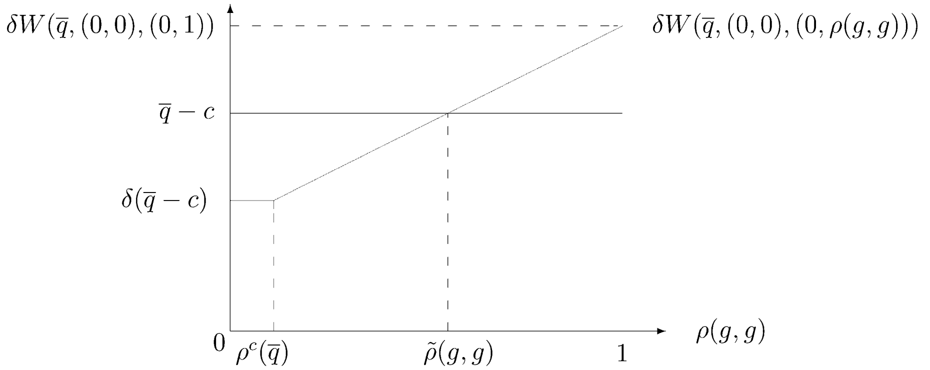

FIGURE 1

EXISTENCE OF AN MSCE IN WHICH $\tilde{\rho}(g, g) \in (0, 1)$

receivers anticipate that the optimistic sender waits. On the basis of A2 and Lemma 2, it is easy to see that there exists then a unique $\tilde{\rho}(g, g)$ that makes them indifferent between investing and waiting. This is depicted in Figure 1.

We now explain why the optimistic sender wants to wait given that the remaining optimistic receivers invest with probability $\tilde{\rho}(g, g)$. Consider therefore the following lemma (and its first corollary).

LEMMA 3. $\forall\, (\sigma_1, \rho_1)$, $\delta W(q, \sigma_1, \rho_1) - (q - c)$ *is decreasing.*

PROOF. See Gossner and Melissas (2003). Lemma 3 is illustrated in Figure 2.

Suppose player j anticipates that $\Theta = G$ with some probability q. As before, Figure 2 shows the existence of a unique $\tilde{\rho}(\cdot)$ where the gain of investing equals the

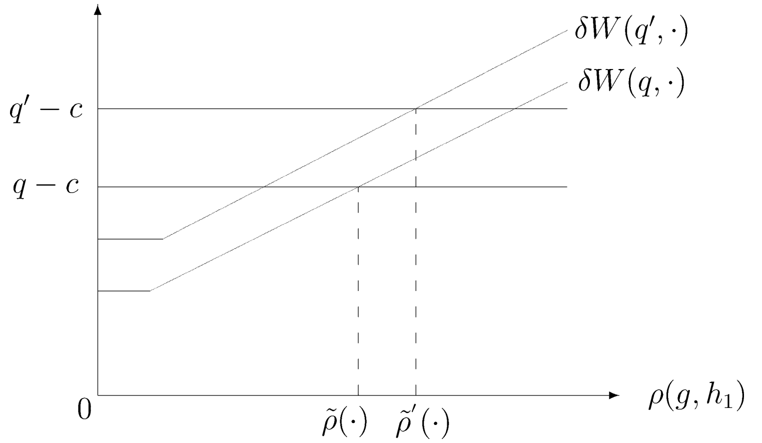

FIGURE 2

THE EFFECT OF A CHANGE IN q ON $q - c$ AND $W(\cdot)$

gain of waiting. Suppose now that for some exogenous reason, player j becomes "more optimistic" in the sense that she now anticipates that $\Theta = G$ with probability $q' > q$. An increase in q shifts the gain of waiting upward for two different reasons: (i) It increases the likelihood that $\Pr(G \mid q, k, a_i) > c$ and thus that player j will get a nonzero expected utility and (ii) it increases her expected gain of investing whenever player j does so. However, the presence of δ in front of $W(q, \cdot)$ (and not in front of $q - c$) dampens this increase in $\delta W(q, \cdot)$, which explains Lemma 3.

COROLLARY 2: *Suppose the sender and the pessimistic receivers wait* (*i.e.*, $\sigma(b, \hat{s}_i) = \sigma(g, \hat{s}_i) = \rho(b, \hat{s}_i) = 0$). *Then*, $\tilde{\rho}(g, \hat{s}_i)$ *is increasing in* $\Pr(G \mid g, \hat{s}_i)$.

The corollary is also illustrated in Figure 2: As the upward shift of the gain of investing dominates the one of the gain of waiting, $\tilde{\rho}(\cdot)$ must increase to make an optimistic receiver indifferent between investing and waiting.

We now know enough to understand why the optimistic sender wants to wait given that $\Pr(G \mid g, g) = \bar{q}$ and that all optimistic receivers invest with probability $\tilde{\rho}(g, g)$. Two different reasons lie at the root of this finding: The first one is due to the fact that the sender observes k^s instead of k, the second one is due to the fact that $p < \bar{q}$. To illustrate the first reason, suppose the sender's posterior probability that $\Theta = G$ equals the one of the optimistic receivers. One can think of the statistics k and k^s as follows. Let the n^r optimistic receivers invest with probability $\tilde{\rho}(\cdot)$. Next, construct k as follows: If player j invested,[12] $k = k^s - 1$, otherwise $k = k^s$. Hence, k^s is a sufficient statistic for k and, thus, player i's gain of waiting cannot be lower than player j's. To illustrate the second reason, suppose that if the sender waits, she observes k instead of k^s. Call a the probability with which the optimistic receivers must invest such that $p - c = \delta W(p, (0, a))$ (i.e., such that an optimistic sender is indifferent between investing and waiting). As $\bar{q} > p$, from Corollary 2 we know that $\tilde{\rho}(g, g) > a$. From Corollary 1, this implies that $p - c < \delta W(p, (0, \tilde{\rho}(g, g)))$.

COROLLARY 3. *Under A2*, $q - c < \delta W(q, (0, 0), (0, 1))$.

PROOF. A2 states, among other things, that $\bar{q} - c < \delta W(\bar{q}, (0, 0), (0, 1))$. From Lemma 3, we know that the downward shift of the gain of investing dominates the one of the gain of waiting. ■

In words, Corollary 3 states that if a player who possesses the highest possible posterior faces a positive option value of waiting, then this will also be true for all less optimistic ones.

PROPOSITION 2. *There does not exist an MSCE in which the optimistic sender, after having sent an unfavorable message, gets a payoff strictly higher than* $p - c - \epsilon$.

PROOF. See Gossner and Melissas (2003). As the optimistic sender "lied," she suffers an ϵ-reputational cost. Thus, if she invests, she gets $p - c - \epsilon$. If she waits, she

[12] Remember that player j is an optimistic receiver who is indifferent between investing and waiting and who, therefore, invests with probability $\tilde{\rho}(\cdot)$.

gets $\delta W(p, \tilde{\rho}_1) - \epsilon$. Hence, if her payoff strictly exceeds $p - c - \epsilon$, this means that she strictly prefers to wait. Suppose there exists an MSCE in which $\tilde{\sigma}(g, b) = 0$. As she sent an unfavorable message, she is the most "optimistic" player in our economy (i.e., $\Pr(G \mid b, \hat{s}_i = b) < \Pr(G \mid g, \hat{s}_i = b) \leq p$). As we restrict attention to *monotone* strategies (in particular, this implies that time-one investment probabilities must weakly increase in time-one posteriors), $\tilde{\rho}(g, b) \leq \tilde{\sigma}(g, b) = 0$. Clearly, this cannot be an MSCE as the optimistic sender, anticipating that no receiver will invest at time one, then strictly prefers to invest. In our companion paper, we prove that if the optimistic sender sends $\hat{s}_i = b$, there exists a unique MSCE in which $\tilde{\sigma}(g, b) > 0$. This implies that her payoff can then not exceed $p - c - \epsilon$.

The explanation above also underscores our need to focus on monotone strategies. Lemma 3 and Corollary 2 already establish that, in equilibrium, the time-one investment probabilities of the receivers (weakly) increase in their time-one posteriors. However, consider a candidate continuation equilibrium in which $\tilde{\rho}(g, b) \in (0, 1)$ and in which the optimistic sender, despite being the most "optimistic" player in the economy, strictly prefers to wait on the grounds that she observes k^s instead of k. Lemma 3 and Corollary 2 are not sufficient to rule out those kind of nonmonotone candidate continuation equilibria. We decided not to study nonmonotone equilibria in this article as we would not expect them to constitute a natural focal point of our game. More research is needed to investigate their existence and their welfare properties.

PROPOSITION 3. *If the investment generates a high surplus and if* $\Pr(G \mid s_j = b, \hat{s}_i = g) = 1/2$, *there exist two (and only two) MSCEs. In the first one, the optimistic receivers invest with probability* $\tilde{\rho}(g, g) \in (0, 1)$, *whereas the other players wait. In the second one, the optimistic sender together with all (optimistic and pessimistic) receivers invest at time one.*

PROOF. See Gossner and Melissas (2003). As mentioned above, if $\Pr(G \mid b, g) = 1/2$, this means that (i) the sender truthfully announced that she is an optimist and (ii) receivers compute their posteriors under the assumption of truthful revelation. For the same reasons as the ones explained above, there exists an MSCE in which only the optimistic receivers randomize at time one. As the investment generates a high surplus, at time one both the optimistic and the pessimistic receivers face a positive gain of investing. Suppose player j anticipates that everyone invests at time one. Player j knows that the sender is an optimist. Thus, she does not expect to learn something about the sender's type by observing her time-one action. Hence, player j only wants to wait to learn something about the other receivers' types. However, the other receivers, independently of their types, also invest at time one. Hence, player j cannot learn by waiting and, because of discounting, prefers to invest at time one.

Note that in this MSCE, all receivers possess some public (i.e., the favorable message sent by player i) and some private information (i.e., their signals). All receivers, independently of their signals, rely on the public information by investing at time one. This behavior is identical to the one followed by the players

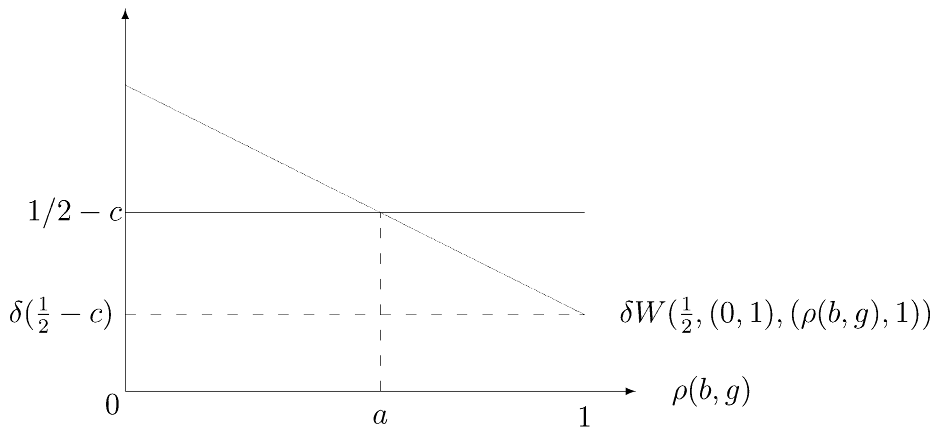

FIGURE 3

AN UNSTABLE CONTINUATION EQUILIBRIUM WHEN ONLY PESSIMISTIC RECEIVERS RANDOMIZE

inside an informational cascade in BHW's (1992) and Banerjee's (1992) models. In those models, all players also possess some public (i.e., the action(s) of the first mover(s)) and private information (i.e., their signals) and, independently of their signals, adopt the same action. Therefore, we call the MSCE in which all receivers invest at time one an informational cascade. Chamley (2004a) has shown that this informational cascade does not hinge on our use of a binomial distribution. Rather, it can be recovered under a wide range of distributional assumptions.

The reader may wonder why there does not exist a third MSCE in which only the pessimistic receivers randomize. The answer is simple: That continuation equilibrium is not stable. To understand this, consider Figure 3.

Suppose player j is a pessimistic receiver who believes the sender to be optimistic. Figure 3 depicts player j's gain of investing and her gain of waiting as a function of $\rho(b, g)$. If $\rho(b, g) = 0$, at time two player j will learn how many optimists are present in the economy and her gain of waiting is maximal. If $\rho(b, g) = \rho(b, g) = 1$, all receivers, independent of their types, invest at time one, and player j's gain of waiting is minimal. Figure 3 reveals the existence of a continuation equilibrium in which all pessimistic receivers invest with probability a. More important, the figure also shows that player j's gain of waiting is decreasing in $\rho(b, g)$. This is intuitive: When only pessimistic receivers randomize (whereas the optimistic receivers invest), the act of waiting becomes informative. The higher $\rho(b, g)$, the harder it is to infer n on the basis of k, and the lower a player's gain of waiting. As player j's gain of waiting is decreasing in $\rho(b, g)$, from Figure 3 it is clear that a small increase (decrease) in $\rho(b, g)$ induces player j to increase (decrease) her equilibrium investment probability from a to one (a to zero). Hence, that equilibrium is unstable.

4. CHEAP TALK

We now analyze player i's incentives to truthfully reveal her private information at time zero. One may think about player i in two ways. First, one may interpret player i as a "guru" whose opinion concerning investment matters is often asked by the media. Second, given our assumptions one would want to introduce an opinion poll (instead of just interviewing one player) at time zero. Unfortunately, analytical results are harder to get when one introduces other players at time zero. Therefore, one can also interpret our model as one explaining "the economics of opinion polls" under the simplifying assumption that the size of the opinion poll equals one. We first state and prove the following "negative" result.

PROPOSITION 4. *For low-surplus projects, there exists a unique MSPBE. In that equilibrium, the optimistic and the pessimistic senders send* $\hat{s}_i = g$. *This MSPBE is supported by the out-of-equilibrium belief that if* $\hat{s}_i = b$, *the sender is a pessimist.*

PROOF. The proposition is proven in two different steps. First, we prove that $\sigma^*(b, h_0)$ must be equal to $\sigma^*(g, h_0)$. Next, we explain why $\sigma^*(b, h_0) = \sigma^*(g, h_0) = 1$. The proof of the first step appears below. The proof of the second step, which is less insightful, can be found in the appendix. We decided to follow this "two-step procedure" to better highlight the role played by the ϵ-reputational cost in our model.

Suppose there exists an MSPBE in which $\sigma^*(g, h_0) > \sigma^*(b, h_0)$. This can only be an equilibrium if the pessimistic sender does not want to deviate, that is, if

$$E(U_i \mid s_i = b, \hat{s}_i = b) \geq E(U_i \mid s_i = b, \hat{s}_i = g)$$

If the sender sends "I am a pessimist," in our companion paper we have proven that our continuation game is then characterized by a unique MSCE in which $\sigma^*(g, b) = 1$ and $\rho^*(g, b) \in [0, 1)$. If the sender sends "I am an optimist," $\Pr(G \mid g, g) > p$ and from Proposition 1, we know that in the continuation game the sender and the pessimistic receivers wait, whereas the optimistic receivers invest with probability $\rho^*(g, g) \in (0, 1)$. We now argue that $\rho^*(g, b) < \rho^*(g, g)$. If $\rho^*(g, b) = 0$, it trivially follows that $\rho^*(g, b) < \rho^*(g, g)$. Therefore, suppose that $\rho^*(g, b) > 0$. In that case, both probabilities are solutions of the following system of two equations:

$$\delta W(\Pr(G \mid g, b), (0, 1), (0, \rho^*(g, b))) - (\Pr(G \mid g, b) - c) = 0 \tag{5}$$

$$\delta W(\Pr(G \mid g, g), (0, 0), (0, \rho^*(g, g))) - (\Pr(G \mid g, g) - c) = 0$$

Suppose Equality (5) is satisfied. From Lemma 3 then follows that

$$\delta W(\Pr(G \mid g, g), (0, 1), (0, \rho^*(g, b))) - (\Pr(G \mid g, g) - c) < 0$$

In Gossner and Melissas (2003, Lemma 5), it is proven that

$$\delta W(\Pr(G \mid g, g), (0, 0), (0, \rho^*(g, b))) \leq \delta W(\Pr(G \mid g, g), (0, 1), (0, \rho^*(g, b)))$$

This is intuitive: A receiver's gain of waiting cannot decrease if the sender chooses a more informative time-one strategy. Hence,

$$\delta W(\Pr(G \mid g, g), (0, 0), (0, \rho^*(g, b))) - (\Pr(G \mid g, g) - c) < 0$$

and from Lemma 2 then follows that $\rho^*(g, b) < \rho^*(g, g)$. From Corollary 1, we know that this implies that

$$\delta W(1 - p, (0, \rho^*(g, b))) < \delta W(1 - p, (0, \rho^*(g, g)))$$

The left-hand side of the inequality above represents $E(U_i \mid s_i = b, \hat{s}_i = b)$, whereas the right-hand side represents $E(U_i \mid s_i = b, \hat{s}_i = g) + \epsilon$. Hence, in the absence of an ϵ-reputational cost $E(U_i \mid g, b) < E(U_i \mid g, g)$, which contradicts the necessary condition we identified earlier. As ϵ is sufficiently close to zero, the pessimistic sender still strictly prefers to send "I am an optimist" to "I am a pessimist," and, thus, for low-surplus projects, no information can be transmitted through words. ■

Intuitively, there does not exist an MSPBE in which $\sigma^*(b, h_0) < \sigma^*(g, h_0)$ because if player i were to send an unfavorable message, this reduces the optimistic receivers' gain of investing and consequently the equilibrium probability $\rho^*(g, \cdot)$. As it becomes then more difficult for the sender to infer n out of k, this reduces the sender's gain of waiting.

The intuition why $\sigma^*(b, h_0) = \sigma^*(g, h_0) = 1$ is based on our ϵ-reputational cost. As messages do not affect posteriors, the optimistic sender cannot influence her gain of waiting. To avoid paying ϵ, she thus strictly prefers to send $\hat{s}_i = g$. The pessimistic sender knows that $\sigma^*(g, h_0) = 1$. As argued above, if she sends $\hat{s}_i = g$, she learns more (about the receivers' types) than by sending $\hat{s}_i = b$ (note, however, that this will be at the expense of her reputation). As $\epsilon \to 0$, she also strictly prefers to send $\hat{s}_i = g$ instead of $\hat{s}_i = b$.

Note that Proposition 4 fundamentally relies on the assumption that players can wait and observe the period-one investment decisions. If players were not allowed to observe past investment decisions, our game would be characterized by a unique PBE in which $\sigma^*(g, h_0) = 1$ and $\sigma^*(b, h_0) = 0$. The intuition is simple: If the sender is optimistic she will, independent of her message, invest in the first period. If she is pessimistic she will, independent of her message, not invest. Hence, to save on the ϵ-reputational cost, a sender strictly prefers to truthfully report her type. Hence, Proposition 4 shows how the credibility of cheap talk statements can be adversely affected when players can learn through actions. As we mentioned in our introduction, the literature on social learning (see, among others, Banerjee, 1992; BHW, 1992; Chamley and Gale, 1994; Chamley, 2004a) assumes that information only gets revealed through actions. As those models are void of any competition effects, some economists wonder why information should not be revealed through words.[13] Proposition 4 thus provides a justification for the "ad hoc" omission of a cheap-talk communication channel in many herding models. This article also possesses a more "positive" result, which is summarized below.

PROPOSITION 5. *For high-surplus projects, our game is characterized by two MSPBEs: a pooling and a separating one. In the separating equilibrium, all receivers, independent of their types, invest at time one if $\hat{s}_i = g$. If $\hat{s}_i = b$, the optimistic receivers invest with probability $\rho^*(g, b)$, whereas the remaining players wait. In the pooling equilibrium, both sender's types send $\hat{s}_i = g$. The pooling equilibrium is supported by the out-of-equilibrium belief that if $\hat{s}_i = b$, the sender is a pessimist.*

PROOF. The existence of a separating equilibrium is proven below. The existence of a pooling equilibrium is proven in the appendix. Finally, in the appendix, we also prove the nonexistence of an MSPBE in which $\sigma^*(b, h_0) < \sigma^*(g, h_0)$.

Suppose the investment project is a high surplus one (i.e., $c \leq 1/2$) and that all receivers revise their posteriors under the assumption that $\sigma^*(b, h_0) = 0$ and that $\sigma^*(g, h_0) = 1$. Consider first the optimistic sender. From Proposition 2, we know that if she deviates and sends $\hat{s}_i = b$, her payoff cannot exceed $p - c - \epsilon$. If she sends $\hat{s}_i = g$, from Proposition 3, we know that there exists a continuation equilibrium in which all receivers, along with the optimistic sender, invest at time one. Hence, *absent the ϵ-reputational cost*, an optimistic sender is indifferent between the two messages. If she prefers not to be caught "lying," she strictly prefers to truthfully report her signal. Consider now the pessimistic sender. If she sends $\hat{s}_i = b, c \leq \Pr(G \mid g, \hat{s}_i = b) = 1/2$. We now argue that $\rho^*(g, b) > 0$ if $c < 1/2$. As all receivers know s_i at time one, no additional information (about the sender's type) can be learned through the observation of a_i. Therefore, a receiver's gain of waiting is independent of σ_1.[14] Hence, if $\Pr(G \mid g, b) = 1/2 > c$,

$$\delta W\left(\frac{1}{2}, (0, 1), (0, 0)\right) = \delta W\left(\frac{1}{2}, (0, 0), (0, 0)\right) = \delta\left(\frac{1}{2} - c\right) < \frac{1}{2} - c$$

From Figure 1, we know there exists then a unique $\rho^*(g,\ b) > 0$ such that an optimistic receiver is indifferent between investing and waiting. From Corollary 1 follows that

$$E(U_i \mid s_i = b, \hat{s}_i = b) = \delta W(1 - p, (0, \rho^*(g, b))) > 0, \quad \forall c < \frac{1}{2}$$

[13] For example, Zwiebel (1995, p. 16) wrote:

> Relative performance evaluation also justify agents' unwillingness to share information, an issue that is problematic in many herding models.

[14] See Gossner and Melissas (2003, Lemma 10) for a formal proof.

If the pessimistic sender deviates and sends $\hat{s}_i = g$, all receivers, independent of their types, invest at time one. As the sender does not receive any payoff-relevant information, she will not invest and $E(U_i \mid s_i = b, \hat{s}_i = g) = -\epsilon$. As

$$E(U_i \mid s_i = b, \hat{s}_i = b) > 0 > E(U_i \mid s_i = b, \hat{s}_i = g) \text{ whenever } c < \frac{1}{2}$$

a pessimist strictly prefers to reveal her unfavorable information. ■

The intuition behind our pooling equilibrium (in which both sender's types send the message $\hat{s}_i = g$) is identical to the one we explained above. In words, a separating equilibrium is fundamentally driven because: (i) both sender's types face different opportunity costs of waiting and (ii) sending a favorable message creates an informational cascade. An optimist believes the investment project is good. For her "time is money" and she is only willing to postpone her investment plans (with probability one) if pessimists do not invest *and* if optimists invest with a relatively high probability. Unfortunately these two aims cannot be simultaneously achieved by any of the two messages. Therefore, in the presence of an ϵ-reputational cost, she strictly prefers to send $\hat{s}_i = g$. A pessimist believes the investment project is bad. She is unwilling to invest unless she observes "relatively many" optimists investing at time one. If the pessimist were to deviate and send a favorable message, an informational cascade would occur, she wouldn't receive any payoff-relevant information, and she would get zero. Hence, it is the informational cascade that ultimately induces a pessimist to send an unfavorable message. If $\rho^*(b, h_1)$ would always be equal to zero (as is the case for low-surplus projects), a pessimist would never want to send a negative message because—if this message were to be believed—this would reduce $\rho^*(g, h_1)$.

Observe that Proposition 5 also stresses the importance of the informational cascade to elicit private information. There only exist two MSPBEs. Thus, there does not exist an MSPBE in which $\sigma^*(b, h_0) < \sigma^*(g, h_0)$ and in which $(\rho^*(b, g), \rho^*(g, g)) \neq (1, 1)$.

So far, we assumed that the sender always possesses private information. In Gossner and Melissas (2003), we allowed for an uninformed sender, in the sense that $s_i \in \{b, \phi, g\}$. If $s_i = \phi$, the sender's signal is completely uninformative. We assumed that $\Pr(s_i = \phi \mid \cdot) = \epsilon$ (where $\epsilon > 0$ and $\epsilon \to 0$) and showed the existence of a semiseparating equilibrium in which the pessimistic and the uninformed sender send the same message (say, message $\hat{s}_i = \phi$) and the optimistic one sends message $s_i = g$. The intuition is similar to the one behind Proposition 5: The pessimistic and the uninformed sender do not want to send the message $\hat{s}_i = g$, as this triggers an informational cascade. The optimistic sender—independent of her message—invests at time one and prefers to report truthfully for reputational reasons. Hence, one should not interpret Proposition 5 as follows: "Informational cascades induce all possible types of players to truthfully reveal their private information." Instead, Proposition 5 should be interpreted as: "Informational cascades put an upper limit above which some types of players don't want to misrepresent their information."

5. SOME NORMATIVE AND POSITIVE IMPLICATIONS

5.1. *Should We Subsidize Investments?* Denote by sub, an investment subsidy granted to each period-one investor. Call $c' \equiv c - sub$. A social planner can, by appropriately choosing sub, alter the amount of learning in two different ways. First, by making it relatively more attractive to invest at time one, she can influence all players' gain of waiting in a favorable way. Second, by setting sub such that $c' \leq 1/2 < c$, she changes the sender's incentives to truthfully reveal her private information (and thus the nature (separating vs. pooling) of the equilibrium played in our game). In a full-fledged welfare study, one should compute the value of sub that maximizes expected welfare. This exercise, however, is lengthy and outside the scope of this article. Rather, in this subsection we assume that $sub \in [-\epsilon, \overline{sub})$ and highlight some advantages and disadvantages of setting $sub \neq 0$. If $sub = -\epsilon$ (where, as above, ϵ represents an arbitrary small, but strictly positive number) this means that the social planner taxes first-period investments. Note that we only allow for a "low" subsidy[15] in the sense that

$$\begin{aligned} sub < \overline{sub} &\equiv \min\{\overline{sub_1}, \overline{sub_2}\}, \qquad \text{where} \\ \overline{sub_1} &\equiv \delta W(\bar{q}, (0,0), (0,1)) - (\bar{q} - c) \quad \text{and} \\ \overline{sub_2} &\equiv c + p - 1 \end{aligned}$$

If $sub < \overline{sub_1}$, this means that the most optimistic type in our model still faces a positive option value of waiting. If $sub < \overline{sub_2}$, this means that $1 - p < c'$. In Gossner and Melissas (2003), we show that $\forall sub \in [-\epsilon, \overline{sub})$, Propositions 4 and 5 are unaffected by the introduction of a first-period subsidy, that is, if $c' > 1/2$, the unique MSPBE is the pooling one, if $c' \leq 1/2$, there exists a separating and a pooling equilibrium.

We first analyze the case in which the first-period subsidy does not change the nature of the played equilibrium. To illustrate our way of working, suppose the investment project is a high surplus one and that players always focus on the separating equilibrium. As mentioned above, in this equilibrium the message of the sender reveals her type, and strategies of period one are given by: after a good message, everyone invests in period 1, after a bad message, optimistic receivers invest with probability $\rho^*(g, b)$, and the remaining players do not invest.

LEMMA 4. *$\forall sub \in [0, \overline{sub})$, $\rho^*(g, b)$ is strictly increasing in sub and $\rho^*(g, b) <$ 1.*

PROOF. See the appendix. The intuition behind Lemma 4 is straightforward. We are considering a separating equilibrium. Thus, after the arrival of an unfavorable message, optimistic receivers know they are the only players in the economy who

[15] We consider an investment subsidy that may be paid to a potentially very large number of firms. In comparison to the investment cost, it is then unlikely that the subsidy would be very important. We do not have in mind a situation in which a government offers a generous subsidy to attract an important investment project (e.g., the subsidy offered by the French Government to attract Eurodisney).

face a positive gain of investing. If an optimistic receiver waits, she forfeits the investment subsidy. Hence, the higher *sub*, the higher a player's cost of waiting. However, in equilibrium, the gain of waiting must equal the cost of waiting, and, thus, the higher *sub*, the higher a player's gain of waiting (and from Figure 1, we know that this requires a higher $\rho^*(g, b)$).

$Wel(g, sub, sep)$ ($Wel(b, sub, sep)$) denotes the expected payoffs (net of the subsidies received) of the optimistic (pessimistic) players given the first-period subsidy and given that all players focus on the separating equilibrium. For the optimistic players, one has

$$Wel(g, sub, sep) = \frac{N}{2}(p - c + sub) - \left(\frac{1}{2}2p(1-p)(N-1)\rho^*(g, b) + \frac{1}{2}[(p^2 + (1-p)^2)(N-1) + 1]\right)sub$$

The first term is given by the expected number of optimists multiplied by their expected utilities. The second is the expected number of optimistic players who invest in period one[16] times the subsidy that is paid to them. This last expression simplifies to

$$Wel(g, sub, sep) = \frac{N}{2}(p - c) + (N-1)p(1-p)(1 - \rho^*(g, b))sub \tag{6}$$

Observe that the second term is strictly positive whenever $sub > 0$. This finding implies that, from a welfare point of view, a strictly positive subsidy is better (insofar as the optimistic players are concerned) than no subsidy at all. From Lemma 4, we know that $(1 - \rho^*(g, b))sub$ (and thus also $Wel(g, sub, sep)$) need not be monotonic in *sub*. This is intuitive: An increase in *sub* increases an optimist's gain of waiting, but also reduces the probability that an optimist will wait and effectively benefit from a more informative signal. For pessimists, one has

$$\begin{aligned} Wel(b, sub, sep) = {} & (N-1)p(1-p)\left(\frac{1}{2} - c\right) \\ & + \frac{1}{2}\big[(p^2 + (1-p)^2)(N-1)\delta W(\Pr(G \mid b, s_i = b), (0, 1), \\ & (0, \rho^*(g, b))) + \delta W(1 - p, (0, \rho^*(g, b)))\big] \end{aligned} \tag{7}$$

The first term corresponds to the expected welfare for pessimistic receivers given an optimistic sender. Similarly, the first term between square brackets corresponds to the expected welfare of all pessimistic receivers given a pessimistic sender. The second term between square brackets corresponds to the expected utility of

[16] With probability $1/2$, the sender is pessimistic, in which case $2p(1-p)(N-1)$ optimistic receivers invest at time one with probability $\rho^*(g, b)$; with probability $1/2$, the sender is optimistic, in which case $(p^2 + (1-p)^2)(N-1) + 1$ optimistic players (= conditional expected number of optimistic receivers plus the optimistic sender) invest at time one with probability one.

the pessimistic sender. From Lemmas 2 and 4 and Corollary 1, it follows that $Wel(b, sub, sep)$ cannot decrease in sub. This is also intuitive: The higher sub, the higher $\rho^*(g, b)$, and, as explained in Section 3, this cannot decrease the expected utilities of the pessimistic players. Total social welfare equals

$$Wel(sub, sep) = Wel(g, sub, sep) + Wel(b, sub, sep)$$

Suppose now that all players, independently of the surplus generated by the project, focus on the pooling equilibrium. From above, we know that both senders' types then send the message $\hat{s}_i = g$, that optimists invest with probability $\rho^*(g, g)$, and that pessimists do not invest. Note that receiving the message $\hat{s}_i = g$ in the pooling equilibrium is informationally different from receiving the same message in the separating one (and, more importantly, leads to a different behavior in the continuation game). To avoid confusion, in this subsection we denote by $\rho^*(g, h_1)$ (respectively, $\rho^*(g, g)$), the probability with which all optimists invest at time one in the pooling (respectively, separating) equilibrium after having received a favorable message. Here again, we estimate the social welfare separately for optimists and for pessimists (total welfare is denoted by $Wel(sub, pool)$). For optimists, this writes:

$$Wel(g, sub, pool) = \frac{N}{2}(p - c) + \frac{N}{2}\big(1 - \rho^*(g, h_1)\big)sub \tag{8}$$

For pessimists, we have:

$$Wel(b, sub, pool) = \frac{N}{2}\delta W\big(1 - p, (0, \rho^*(g, h_1))\big) \tag{9}$$

LEMMA 5. *$\forall sub \in [0, \overline{sub})$, $\rho^*(g, h_1)$ is strictly increasing in sub and $\rho^*(g, h_1) < 1$.*

PROOF. See the appendix. The intuition is similar to the one behind Lemma 4. As above, $Wel(g, sub, pool)$ need not be monotonic in sub, whereas $Wel(b, sub, pool)$ cannot decrease in sub. Our main result is summarized below.

PROPOSITION 6. *If the subsidy does not alter the nature of the played equilibrium, any $sub \in (0, \overline{sub})$ is (strictly) better (for welfare) than no subsidy at all. The relationship between welfare and sub need, however, not be monotonic.*

Proposition 6 is not very surprising: Because of the information externality the social benefit of investing at time one exceeds the private one. Hence, a social planner fixes $sub > 0$ to close the gap between both benefits. A similar result is also present in Doyle (2002). However, it would be premature to conclude that—in the presence of information externalities—investments must always be subsidized as the example below suggests.

Suppose $c = 1/2$ and that our players focus on the separating equilibrium. We now show that the social planner can increase welfare by imposing an arbitrarily small, but strictly positive, investment tax (i.e., $sub = -\epsilon$). We first compute $Wel(0,\ sep)$. Observe that in the separating equilibrium $\Pr(G \mid s_j = g, \hat{s}_i = b) = 1/2 = c$, and thus there exists a PBE in which $\rho^*(g,\ b) = 0$. Hence, from Equation (6) follows that

$$Wel(g, 0, sep) = \frac{N}{2}(p - c) \tag{10}$$

As $\rho^*(g, b) = 0$,

$$\delta W(\Pr(G \mid b, s_i = b), (0, 1), (0, 0)) = \delta W(1 - p, (0, 0)) = 0$$

and from Equation (7) we know that

$$Wel(b, 0, sep) = (N - 1)p(1 - p)\left(\frac{1}{2} - c\right) = 0 \tag{11}$$

Adding (10) and (11), one has

$$Wel(0, sep) = \frac{N}{2}(p - c). \tag{12}$$

This is intuitive: If $\hat{s}_i = g$, pessimists invest at time one and get a zero payoff. If $\hat{s}_i = b$, $\rho^*(g, b) = 0$ and our pessimistic players also get a zero payoff. Hence, if $c = 1/2$, total welfare is only determined by the expected utilities of the optimistic players. If $\hat{s}_i = g$, all optimists invest at time one. If $\hat{s}_i = b$, optimistic receivers do not invest, but nonetheless obtain the same payoff (i.e., zero) as the one they would obtain if they were to invest at time one. Stated differently, unconditionally investing at time one is—for an optimist—payoff equivalent to the alternative strategy in which she only invests if $\hat{s}_i = g$. Thus, an optimist gets $p - c$ and, in expected terms, half of the population is optimistic. Thus, welfare equals $N/2(p - c)$.

If $sub = -\epsilon$, $c' > 1/2$ and the unique MSPBE is the pooling one. As $\epsilon \to 0$,

$$Wel(g, -\epsilon, pool) \to \frac{N}{2}(p - c) \text{ and } Wel(b, -\epsilon,\ pool) = \delta W(1 - p, (0, \rho^*(g, h_1)))$$

As $\rho^*(g, h_1) > \rho^*(g, b) = 0$, pessimists benefit from a more informative statistic in the pooling equilibrium and thus $Wel(0, sep) < Wel(-\epsilon, pool)$. Our main insight is summarized below.

PROPOSITION 7. *An investment tax can—by altering the nature of the played equilibrium—(strictly) increase welfare.*

In the analysis above, we restricted ourselves to the case in which $c = 1/2$. However, it should be clear that Proposition 7 is crucially driven by the fact that when c is close to $1/2$ (and $c \leq 1/2$), the expected utility of a pessimist hardly exceeds zero in the separating equilibrium. In our introduction, we explained why our last insight is not present in Doyle (2002).

5.2. *How Does the Sender's Ability Influence Her Incentives for Truthful Revelation?* So far, we assumed that the sender was "as able" as the receivers in the sense that all players possess a signal of the same precision. One may find it more natural to endow player i with a more precise signal. After all, in our model, she can be interpreted as a guru and people typically think of them as being better informed. There is a straightforward way to allow for a better informed sender. Let us assume that player i's signal is drawn from the distribution: $\Pr(g \mid G) = \Pr(b \mid B) = r$ and $\Pr(b \mid G) = \Pr(g \mid B) = 1 - r$ (where $1 > r > p$). The higher r, the "smarter" or the better informed the sender. Our main result is summarized below.

PROPOSITION 8. $\forall c \in (1 - p, \min\{p, \frac{(1-p)r}{(1-p)r + p(1-r)}\})$, $\exists$ *a separating equilibrium. This range of parameter values cannot decrease in the precision of the sender's signal.*

PROOF. An MSCE in which $\tilde{\rho}(b, g) = \tilde{\rho}(g, g) = 1$ exists only if $\Pr(G \mid b, \hat{s}_i = g) \geq c$. This posterior probability is now computed as:

$$\Pr(G \mid b, \hat{s}_i = g) = \frac{\Pr(G, \hat{s}_i = g \mid b)}{\Pr(\hat{s}_i = g \mid b)} = \frac{(1-p)r}{(1-p)r + p(1-r)} > \frac{1}{2}$$

Using a reasoning identical to the one we outlined above, one can check that if $c \in (1 - p, \frac{(1-p)r}{(1-p)r+p(1-r)})$, there exists a separating equilibrium. ■

The intuition behind Proposition 8 is simple. As we showed in Proposition 5, a separating equilibrium only exists if the sender can make the pessimists change their minds. Proposition 8 therefore rests on the intuitive idea that the "smarter" the sender (or the more precise her private information), the "easier" it will be for her to make the pessimists change their minds. If the sender cannot convince the remaining pessimists to invest at time one (either because the sender is commonly perceived to be "stupid" or because the investment project only generates a low surplus), then she does not want to reveal any unfavorable information because this will worsen her second-period inference problem.

6. CONCLUSIONS

In this article, we introduced cheap talk in an investment model with information externalities. We first showed that for low-surplus projects, the unique MSPBE is the pooling one. This is because a pessimist is reluctant to divulge her bad information as this worsens her second-period inference problem. For high-surplus projects, however, there exists a separating equilibrium: As a pessimist does not learn anything upon observing an informational cascade (which occurs whenever the sender sends a favorable message) revelation of bad information is compatible with maximizing behavior. A subsidy on low-surplus projects increases welfare, *provided the subsidy does not turn a low-surplus project into a high-surplus one*. Without an adequate equilibrium selection theory, one cannot appraise the welfare consequences of a policy aimed at subsidizing high-surplus projects. Finally, we argued that "smart" people have more incentives to truthfully reveal their private information than "stupid" ones.

The reader must bear in mind that we only introduced cheap talk in an endogenous-queue setup. More research is thus needed to check the robustness of exogenous-queue herding models to the introduction of cheap talk. In our model, one should think about the sender as a famous investor who is being interviewed by the media. We believe it would be equally interesting to consider a setup in which many players have access to the communication channel through words. In particular, we have two interpretations in mind. First, one could model "the economics of opinion polls" in which a subset of the population is asked to simultaneously send a message to all players in the economy.[17] Second, one could model "the economics of business lunches" in which a subset of the population meet and discuss the investment climate prior to the first investment date (the outcome of the discussion is not divulged to the other players in the economy). We also believe this to constitute an interesting topic for future research.

APPENDIX

PROOF OF PROPOSITION 4. As we consider here low-surplus investment projects, $c > 1/2$. From Proposition 2 we know that

$$E(U_i \mid s_i = g, \hat{s}_i = b) = p - c - \epsilon < E(U_i \mid s_i = g, \hat{s}_i = g) = \max\{p - c, \delta W(\cdot)\}$$

Thus, $\sigma^*(g, h_0) = 1$. In the article we already proved that there does not exist an MSPBE in which $0 \leq \sigma^*(b, h_0) < 1$. We now show the existence of an MSPBE in which $\sigma^*(b, h_0) = \sigma^*(g, h_0) = 1$. If the pessimistic sender deviates and sends $\hat{s}_i = b$, then we assume that receivers believe with probability one that the sender is a pessimist. Hence, $\Pr(G \mid g, b) = 1/2 < c$, and $E(U_i \mid s_i = b, \hat{s}_i = b) = 0$. If $\hat{s}_i = g$, $\Pr(G \mid g, g) = \Pr(G \mid g)$ (i.e., the posterior of the optimistic receivers is equal to the one of the optimistic sender). As explained in our article, we thus impose the restriction that $\rho^*(g, g) = \sigma^*(g, g)$. Observe also that $\Pr(G \mid b, g) = \Pr(G \mid b) = 1 - p < c$. Thus, $\rho^*(b, g) = \sigma^*(b, g) = 0$. Suppose players j and i are both optimists. As the sender's message is completely uninformative, $\Pr(s_i = g \mid s_j = g) = \Pr(s_j = g \mid s_i = g)$. Hence, if player i observes player j investing (waiting), this is informationally equivalent to player j observing player i investing (waiting). Formally, this insight implies that

$$\delta W(p, (0, \rho(g, g))) = \delta W(p, (0, \rho(g, g)), (0, \rho(g, g)))$$

[17] In contrast to Sgroi (2002), we have in mind a situation in which the sender wants to learn the receivers' private information.

If $\rho(g,g)=\sigma(g,g)=0$,

$$p-c>\delta W(p,(0,0))=\delta W(p,(0,0),(0,0))=\delta(p-c)$$

If $\rho(g,g)=\sigma(g,g)=1$, from Corollary 3 we know that

$$p-c<\delta W(p,(0,0),(0,1))\leq\delta W(p,(0,1),(0,1))=\delta W(p,(0,1))$$

where the second inequality follows from the fact that a receiver's gain of waiting cannot decrease when the sender adopts a more informative time-one strategy.[18] It then follows from Corollary 1 that there exists a unique $\rho^*(g,g)>0$ such that

$$p-c=\delta W(p,(0,\rho^*(g,g)))=\delta W(p,(0,\rho^*(g,g)),(0,\rho^*(g,g)))$$

As $\rho^*(g,g)>0$, it follows from Corollary 1 that

$$E(U_i\,|\,s_i=b,\hat{s}_i=b)=0<\delta W(1-p,(0,\rho^*(g,g)))=E(U_i\,|\,b,g)+\epsilon$$

As $\epsilon\rightarrow 0$, a pessimistic sender strictly prefers to send $\hat{s}_i=g$ instead of $\hat{s}_i=b$. ■

Define $\Delta^r(q,\sigma_1,\rho_1)\equiv\delta W(q,\sigma_1,\rho_1)-(q-c)$ as the difference between player j's gain of waiting and her gain of investing. The lemma below, whose proof can be found in Gossner and Melissas (2003), will often be used in our next proofs.

LEMMA 6. *$\Delta^r(\frac{1}{2},\sigma_1,\rho_1)$ and $\Delta^r(\bar{q},\sigma_1,\rho_1)$ are independent of σ_1.*

The lemma is not unintuitive: As $q=1/2$ or $q=\bar{q}$, this means that receivers learned s_i through $\hat{s}_i$. Hence, receivers do not expect to learn anything (about the sender's type) through her period-one action. Hence, receivers do not care about the sender's time-one strategy.

PROOF OF PROPOSITION 5. We first prove the nonexistence of an MSPBE in which $\sigma^*(b,h_0)<\sigma^*(g,h_0)$. Next, we prove the existence of a pooling equilibrium. The existence of a separating equilibrium is proven in the article.

From the proof of Proposition 4, we know that $\sigma^*(g,h_0)=1$. Suppose there exists an MSPBE in which $0<\sigma^*(b,h_0)<\sigma^*(g,h_0)=1$. $\sigma^*(b,h_0)$ can only be $\in(0,1)$ if $E(U_i\,|\,s_i=b,\hat{s}_i=b)=E(U_i\,|\,s_i=b,\hat{s}_i=g)$. If the pessimistic sender sends $\hat{s}_i=b$,

$$\Pr(G\,|\,b,b)<1-p<c\leq\frac{1}{2}=\Pr(G\,|\,g,b)<p$$

In our companion paper, we have proven that the continuation game is then characterized by a unique MSCE in which $\rho^*(g,b)\in[0,1)$ and $\sigma^*(g,b)=1$ (i.e.,

[18] See Gossner and Melissas (2003, Lemma 5) for a formal proof.

if the optimistic sender deviates and sends $\hat{s}_i = b$, it is optimal for her to invest at time one). If she sends $\hat{s}_i = g$, there are two possibilities: (a) $1 - p < \Pr(G \mid b, g) < c < p < \Pr(G \mid g, g)$ and (b) $1 - p < c \leq \Pr(G \mid b, g) < p < \Pr(G \mid g, g)$.

In case (a), in our companion paper we have proven that the continuation game is then characterized by a unique MSCE in which $\rho^*(b, g) = 0$ and $\rho^*(g, g) \in (0, 1)$. Hence,

$$E(U_i \mid s_i = b, \hat{s}_i = b) = \delta W(1 - p, (0, \rho^*(g, b))) \quad \text{and}$$
$$E(U_i \mid s_i = b, \hat{s}_i = g) = \delta W(1 - p, (0, \rho^*(g, g))) - \epsilon$$

We now prove that $\rho^*(g, g) > \rho^*(g, b)$. If $\rho^*(g, b) = 0$, it trivially follows that $\rho^*(g, g) > \rho^*(g, b)$. Therefore, suppose that $\rho^*(g, b) > 0$. In that case, $\rho^*(g, b)$ and $\rho^*(g, g)$ were "generated" by the following two equalities:

$$\Delta^r(\Pr(G \mid g, b), (0, 1), (0, \rho^*(g, b)) = 0 \tag{A.1}$$
$$\Delta^r(\Pr(G \mid g, g), (0, 0), (0, \rho^*(g, g)) = 0$$

As $\Pr(G \mid g, b) = 1/2$, from Lemma 6, we know that

$$\Delta^r(\Pr(G \mid g, b), (0, 1), (0, \rho^*(g, b)) = \Delta^r(\Pr(G \mid g, b), (0, 0), (0, \rho^*(g, b))$$

As $\Pr(G \mid g, b) < \Pr(g, g)$, from Lemma 3 we know that

$$\Delta^r(\Pr(G \mid g, g), (0, 0), (0, \rho^*(g, b)) < \Delta^r(\Pr(G \mid g, b), (0, 0), (0, \rho^*(g, b)) = 0$$

Hence, for equality (A.1) to be respected it follows from Lemma 2 that $\rho^*(g, g) > \rho^*(g, b)$. But then it follows from Corollary 1 that $\delta W(1 - p, (0, \rho^*(g, g))) > \delta W(1 - p, (0, \rho^*(g, b)))$. As $\epsilon \to 0$, it follows that in case (a) $E(U_i \mid s_i = b, \hat{s}_i = b) < E(U_i \mid s_i = b, \hat{s}_i = g)$, a contradiction.

In our companion paper, we have shown that in case (b) there exists an MSCE in which $\rho^*(b, g) = \sigma^*(g, g) = 0$ and $\rho^*(g, g) \in (0, 1)$. Depending on the values of the exogenous parameters, there may also exist another MSCE in which $\sigma^*(b, g) = 0$ and $\sigma^*(g, g) = \rho^*(b, g) = \rho^*(g, g) = 1$. If players focus on the continuation equilibrium in which $\rho^*(b, g) = 0$ and $\rho^*(g, g) \in (0, 1)$, using a reasoning identical to the one of the paragraph above, we know that the pessimistic sender cannot be indifferent between the two messages. Therefore, suppose players focus on the continuation equilibrium in which $\rho^*(b, g) = \rho^*(g, g) = 1$ (provided this continuation equilibrium exists). In that case,

$$E(U_i \mid s_i = b, \hat{s}_i = b) = \delta W(1 - p, (0, \rho^*(g, b))) \quad \text{and}$$
$$E(U_i \mid s_i = b, \hat{s}_i = g) = -\epsilon$$

As $\delta W(1 - p, (0, \rho^*(g, b))) \geq 0 > -\epsilon$, in case (b) the sender cannot be indifferent between the two messages.

We now prove the existence of a pooling equilibrium. Suppose receivers update their posteriors under the assumption that $\sigma^*(b, h_0) = \sigma^*(g, h_0) = 1$. In the out-of-equilibrium event that $\hat{s}_i = b$, we assume that receivers believe that the sender is a pessimist (with probability one). In our companion paper, we have shown that the continuation game is then characterized by a unique MSCE in which $\sigma^*(b, b) = \rho^*(b, b) = 0$, $\rho^*(g, b) \in [0, 1)$, and $\sigma^*(g, b) = 1$. Therefore,

$$E(U_i \mid s_i = b, \hat{s}_i = b) = \delta W(1 - p, (0, \rho^*(g, b)))$$

If she sends $\hat{s}_i = g$, $\Pr(G \mid b, g) = 1 - p < c < \Pr(G \mid g, g) = p$, and from the Proof of Proposition 4, we know that $\rho^*(b, g) = 0$ and $\rho^*(g, g) \in (0, 1)$. We now show that $\rho^*(g, g) > \rho^*(g, b)$. If $\rho^*(g, b) = 0$, it trivially follows that $\rho^*(g, g) > \rho^*(g, b)$. Therefore, supppose that $\rho^*(g, b) > 0$. In that case, both probabilities are "generated" out of the following two equalities:

$$\text{(A.2)} \qquad \Delta^r\left(\frac{1}{2}, (0, 1), (0, \rho^*(g, b))\right) = 0$$
$$\Delta^r(p, (0, \rho^*(g, g)), (0, \rho^*(g, g)) = 0$$

Succesively applying Lemmas 3 and 6 one has

$$0 = \Delta^r(p, (0, \rho^*(g, g)), (0, \rho^*(g, g)) < \Delta^r\left(\frac{1}{2}, (0, \rho^*(g, g)), (0, \rho^*(g, g))\right)$$
$$= \Delta^r\left(\frac{1}{2}, (0, 1), (0, \rho^*(g, g))\right)$$

Hence, for equality (A.2) to be respected it follows from Lemma 2 that $\rho^*(g, b) < \rho^*(g, g)$. From Corollary 1 (plus the fact that $\epsilon \to 0$) follows that the pessimistic sender strictly prefers to "lie" and send $\hat{s}_i = g$. ■

PROOF OF LEMMA 4. Define $\rho^*(g, b, sub)$ as the probability that ensures the following equality

$$\frac{1}{2} - c + sub = \delta W\left(\frac{1}{2}, (0, 1), (0, \rho^*(g, b, sub))\right)$$

By assumption,

$$\text{(A.3)} \qquad sub < \Delta^r(\Pr(G \mid g, s_i = g), (0, 0), (0, 1))$$

We now show that $\forall\, sub \in [0, \overline{sub})$, $\rho^*(g, b, sub) < 1$. $\rho^*(g, b, sub) = 1$ only if

$$\text{(A.4)} \qquad sub \geq \Delta^r\left(\frac{1}{2}, (0, 1), (0, 1)\right)$$

Inequalities A.3 and A.4 cannot both be satisfied as we can use Lemmas 3 and 6 to construct the following contradiction

$$sub \geq \Delta^r\left(\frac{1}{2}, (0,1), (0,1)\right) > \Delta^r(\Pr(G \mid g, s_i = g), (0,1), (0,1))$$
$$= \Delta^r(\Pr(G \mid g, s_i = g), (0,0), (0,1)) > sub$$

As $\rho^*(g, b, sub) < 1$, it follows from Lemma 2 that $\rho^*(g, b, sub)$ is strictly increasing in *sub*. ■

PROOF OF LEMMA 5. The proof is similar to the one of Lemma 4. Define $\rho^*(g, h_1, sub)$ as the probability that ensures the following equality

$$sub = \Delta^r(p, (0, \rho^*(g, h_1, sub)), (0, \rho^*(g, h_1, sub)))$$

$\forall sub \in [0, \overline{sub})$, $\rho^*(g, h_1, sub) < 1$ as we otherwise run into the following contradiction

$$sub \geq \Delta^r(p, (0,1), (0,1)) > \Delta^r(\Pr(G \mid g, s_i = g), (0,1), (0,1))$$
$$= \Delta^r(\Pr(G \mid g, s_i = g), (0,0), (0,1)) > sub$$

As $\rho^*(g, h_1, sub)$ is always strictly lower than one, it follows from Lemma 2 that $\rho^*(g, h_1, sub)$ is strictly increasing in *sub*. ■

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