Comparative Statics of Information Acquisition and Risk Aversion*

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Abstract: We study how willingness to pay for information depends on risk aversion when a decision maker faces background risk and can acquire information before choosing from a menu of assets. We distinguish investment menus, whose payoffs are procyclical with background wealth, from insurance menus, whose payoffs are countercyclical. Our main results show that the interaction between asset cyclicality and the tail geometry of background risk determines the direction of the comparative statics. When the density of background risk is log-concave, willingness to pay for information decreases with risk aversion for investment menus, whereas with downward-log-convex background risk it increases with risk aversion for insurance menus. The proofs compare the distributions of terminal wealth with and without information and develop new aggregation arguments for state-dependent single-crossing comparisons. We also construct reversals under strictly log-convex tails for investment menus and super-exponential left tails for insurance menus.

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1 Introduction

Information enables economic agents to adapt financial decisions to changing economic circumstances. It shapes both investment decisions (Grossman and Stiglitz 1980) and insurance choices (Rothschild and Stiglitz 1976). When information acquisition is itself a choice, an important question is which agents are most willing to acquire information, and in particular how this willingness depends on their attitudes toward risk.

The answer depends on how information is used. A less risk-averse investor may value information that helps direct exposure toward attractive investment opportunities, whereas a more risk-averse household or firm may value information that helps direct insurance toward adverse circumstances. We ask the following comparative statics question: under what conditions does greater risk aversion increase or decrease willingness to pay for information?

We study this question in the presence of background risk. Before deciding whether to acquire information or select an asset, the decision maker already faces uncertainty about her initial wealth. She may purchase a Blackwell experiment and then choose an asset contingent on the observed signal. Purchasing information entails a fixed price, which shifts wealth downward in every state, but also permits signal-contingent asset choice. The value of information depends on how these two effects jointly reshape the distribution of terminal wealth.

Our main results show that the direction of the comparative statics depends jointly on the cyclicality of the available assets and the tail geometry of background risk. Investment menus are procyclical: higher-ranked assets shift payoffs toward states in which background wealth is already high. Insurance menus are countercyclical: higher-ranked assets shift payoffs toward states in which background wealth is low. Under log-concave background risk, willingness to pay for information decreases with risk aversion for investment menus. Under downward-log-convex background risk, it increases with risk aversion for insurance menus.

The underlying mechanisms differ across the two environments. With insurance menus, information directs protection toward adverse realizations of background risk. Under downward log convexity, with support unbounded below, paying a fixed price has a relatively limited effect on lower-tail probabilities, while signal-contingent insurance can substantially improve outcomes in the worst states. Greater risk aversion therefore raises the value of information. With investment menus, information permits the decision maker to avoid exposure in the most adverse states while retaining investment opportunities in nearby, less adverse states. Under log-concave tails, the downward wealth translation generated by the information price has a comparatively strong effect on lower-tail probabilities. The resulting balance lowers willingness to pay as risk aversion increases.

The proofs compare the complete distributions of terminal wealth with and without information. We first establish the existence of monotone optimal strategies under a MLR-ordering condition on the experiment. We then derive opposite conditional single-crossing comparisons for investment and insurance menus. The principal technical difficulty is that these comparisons depend on the realization of background wealth and must therefore be aggregated. Our main technical tool is a state-dependent single-crossing aggregation theorem extending the classical preservation result of Karlin and Rubin (1956). It applies directly to investment menus, while insurance menus require combining the same single-crossing principle on the lower region with a direct distributional comparison elsewhere.

We also construct reversals under tail behavior opposite to that supporting the positive results. For investment menus, strict log convexity reverses the log-concave geometry of Theorem 1, and willingness to pay for information may increase with risk aversion. For insurance menus, super-exponentiality provides a counterpart to downward log convexity: whereas downward log convexity keeps $(\log f)'$ bounded above as wealth tends to $-\infty$, super-exponentiality makes it diverge to $+\infty$. The information price can then dominate targeted insurance in the extreme left tail, and willingness to pay may decrease with risk aversion.

The comparative statics determine which agents choose to become informed before making financial decisions. This selection may matter in broader models of financial and insurance markets, where the distribution of information across agents can affect price formation (Grossman and Stiglitz 1980; Van Nieuwerburgh and Veldkamp 2010) and adverse selection (Rothschild and Stiglitz 1976). We do not analyze these equilibrium consequences here; our results instead identify a primitive force governing information acquisition that can be incorporated into such models. We also briefly consider mixed asset menus, comprising both investment and insurance opportunities, to examine how risk aversion affects the type of asset selected, separately from its effect on willingness to pay for information.

A broad literature in macroeconomics and finance emphasizes the economic importance of tail behavior (Rietz 1988; Barro 2006; Gabaix 2012), while the literature on information and risk aversion obtains sign results that depend on the decision problem and payment convention (Gould 1974; Hilton 1981; Freixas and Kihlstrom 1984; Willinger 1989; Nadiminti, Mukhopadhyay, and Kriebel 1996; Kihlstrom 2022). Our results connect these literatures by identifying asset cyclicality and the tail geometry of background risk as joint determinants of the sign of this comparative static within an expected-utility framework.

The remainder of the paper is organized as follows. Section 2 presents motivating examples. Section 3 introduces the model and defines willingness to pay for information. Section 4 develops the main comparative-statics results and the aggregation arguments underlying them. Section 5 constructs reversals under the opposite tail geometry. Section 6 examines the role of the tail assumptions, presents applications to financial options and continuous-state environments with multiple assets, and then considers the distinct asset-selection question raised by mixed menus. Section 7 discusses the related literature. Section 8 concludes. Proofs omitted from the text are collected in the Appendix.

2 Motivating Examples

Before introducing the general model, we illustrate the two opposite comparative statics that motivate the paper. In both examples, information is valuable because it allows the decision maker to condition an asset choice on the realized state. The salient difference is the economic role of the asset. In the first example, information facilitates the selective adoption of a risky investment, whereas in the second it facilitates the selective use of an insurance opportunity. The resulting comparative statics point in opposite directions and suggest that the nature of the available assets is a first determinant of how the value of information varies with risk aversion.

The examples deliberately use two states and CARA utility so that this investment-versus-insurance mechanism can be seen without imposing a tail-shape condition. This distinction is useful for interpreting the main results below. With only two support points, discrete log concavity or log convexity imposes no nontrivial restriction. The log-concavity and downward-log-convexity assumptions in our main theorems become substantively relevant when we move to general background-risk distributions and a broad class of utility functions, where conditional distributional comparisons must be aggregated across wealth states.

Example 1. Suppose that a businessperson with CARA utility $u_r(m) = -e^{-rm}$, $r > 0$, is deciding whether to invest in a technology startup producing an electric car. There are two states of the world: high (the economy goes generally well), and low (the economy tumbles). The payoff from not investing is (k_H, k_L), with $k_H > k_L$. The net payoff from investing is $b_H = H > 0$ in the high state (electric mobility succeeds only in a strong economy), and $b_L = -L$ in the low state, where $L > 0$. Thus, the investment is procyclical. We consider an information structure α that perfectly reveals the state. Let $p \in (0,1)$ denote the prior probability of the low state, and assume that

$$(1-p)H - pL < 0.$$

so that a risk-neutral agent does not invest without information. Under the displayed inequality and $k_H > k_L$, the status quo is optimal for every $r > 0$. Indifference between remaining uninformed and purchasing full information at price I gives

$$I = -\frac{1}{r}\log\left(\frac{(1-p)e^{-r(H+k_H)} + pe^{-rk_L}}{(1-p)e^{-rk_H} + pe^{-rk_L}}\right).$$

Lemma 1 shows that willingness to pay is strictly decreasing in r.

Lemma 1. *Let* $0 < p < 1$*,* $H > 0$*, and* $k_H > k_L$*. Define, for* $r > 0$*,*

$$g(r) = -\frac{1}{r} \ln\left(\frac{(1-p)e^{-r(H+k_H)} + pe^{-rk_L}}{(1-p)e^{-rk_H} + pe^{-rk_L}} \right).$$

Then g *is strictly decreasing on* $(0, \infty)$*.*

Example 2. Suppose now that the businessperson has the same CARA utility and is deciding whether to invest in a company that produces small prefabricated houses that are very cheap to build but are not very desirable. There are two states of the world: high (the economy goes generally well), and low (the economy tumbles).

The payoff from not investing is (k_H, k_L), with $k_H > k_L$. The net payoff from investing is $b_H = -H < 0$ in the high state (the cheap house does not sell well in a strong economy), and $b_L = L > 0$ in the low state (where low-quality housing is in high demand), with $k_L + L < k_H - H$. Thus, the asset is countercyclical and acts as an insurance opportunity. We consider an information structure α that perfectly reveals the state. Let $p \in (0, 1)$ denote the prior probability of the low state, and assume that

$$-(1-p)H + pL < 0,$$

or equivalently

$$p < \frac{H}{H+L}$$

so that a risk-neutral agent does not invest without information. Under the displayed restrictions, there is a unique $\bar{r} > 0$ such that the status quo is optimal without information if and only if $0 < r \le \bar{r}$. For these coefficients, indifference between remaining uninformed and purchasing full information at price I gives

$$I = -\frac{1}{r} \log\left(\frac{(1-p)e^{-rk_H} + pe^{-r(L+k_L)}}{(1-p)e^{-rk_H} + pe^{-rk_L}} \right).$$

Thus willingness to pay equals $g(r)$ in Lemma 2, which is strictly increasing in r.

Lemma 2. *Let* $0 < p < 1$*,* $L > 0$*, and* $k_H > k_L + L$*. Define, for* $r > 0$*,*

$$g(r) = -\frac{1}{r} \ln\left(\frac{(1-p)e^{-rk_H} + pe^{-r(L+k_L)}}{(1-p)e^{-rk_H} + pe^{-rk_L}} \right).$$

Then g *is strictly increasing on* $(0, \infty)$*.*

The examples isolate the cyclicality channel. Information supports greater exposure in the good state in the investment example and greater protection in the bad state in the insurance example. Because the signal fully reveals the state, this channel alone determines the comparative statics among CARA coefficients for which the status quo is optimal without information. Proposition 4 in Appendix A establishes the corresponding distribution-free benchmark: under full revelation, willingness to pay decreases with risk aversion when every asset is an investment and increases when every asset is an insurance.

Imperfect information creates the second ingredient in the main results. Paying for information shifts wealth downward in every state, while noisy signal-contingent asset choice moves probability mass across a cutoff in selected states. The relevant conditional comparisons must therefore be aggregated across background-wealth realizations. Log concavity makes the price effect relatively important in the lower tail for investment menus. Downward log convexity gives the opposite local geometry for insurance menus, while an unbounded lower support rules out a hard-boundary effect for very risk-averse agents. Section 3 now introduces the general model, and Section 6.1 returns to these distinct roles.

3 Environment

This section introduces the decision problem and defines willingness to pay for information. The decision maker faces uncertain initial wealth, may purchase information about its realization, and then selects an asset whose payoff depends on that realization. We subsequently compare decision makers who face the same economic environment but differ in risk aversion.

The agent has uncertain wealth W with CDF F, whose realization is denoted w. We refer to F as the distribution of *background risk*. We assume that $\int |w| \, \mathrm{d}F(w) < \infty$ and that F admits a density $f : \mathbb{R} \to \mathbb{R}_+$. She has a (strictly) increasing utility function over money $u : \mathbb{R} \to \mathbb{R}$, which is F-integrable, absolutely continuous on any bounded interval, and satisfies $\limsup_{m\to\infty} u(m)/m < \infty$.[1] We let U be the set of functions $\mathbb{R} \to \mathbb{R}$ satisfying these properties. Recall that, given $u, v \in U$, v is more *risk-averse* than u if $v = \phi \circ u$ for some increasing and (weakly) concave $\phi : \mathbb{R} \to \mathbb{R}$.

An *asset* describes an extra monetary payoff received by the agent as a function of the realization of w. It is given by an F-integrable map $b : \mathbb{R} \to \mathbb{R}$, where $b(w)$ is the amount paid by the asset when the background risk realization is w. The agent has access to a set of assets B. We assume that B includes the status quo which gives $b(w) = 0$ for every w, and which we denote $\mathbf{0}$. We also assume that B is compact when endowed with the L^1-metric induced by F.[2]

The agent may decide to purchase information prior to choosing an asset. In this

1. A map $u : \mathbb{R} \to \mathbb{R}$ is F-integrable if it is measurable and such that $\int |u| \, \mathrm{d}F < \infty$; the growth condition on u is used to ensure that all integrals involving u that we consider are well defined.

2. That is, we assume that any sequence $(b_n)_{n\in\mathbb{N}}$ in B admits a subsequence $(b'_n)_{n\in\mathbb{N}}$ such that, for some $b \in B$, $\int |b'_n - b| \, \mathrm{d}F \to 0$ as $n \to \infty$.

case, she pays a price $\mu \geq 0$ and gains access to a Blackwell experiment:

$$\alpha = (X, (h_w)_{w \in \mathbb{R}})$$

where X is a finite set of signals, h_w is a probability mass function over X for each $w \in \mathbb{R}$, and the maps $w \mapsto h_w(x)$ are measurable for each $x \in X$. We assume that α is *MLR-ordered*: $X \subset \mathbb{R}$, and the kernel $(w, x) \mapsto h_w(x)$ is TP2, meaning that

$$h_w(x) h_{w'}(x') \geq h_w(x') h_{w'}(x)$$

whenever $w \leq w'$ and $x \leq x'$.[3]

When all signal probabilities are strictly positive, this condition is equivalent to requiring $h_w(x')/h_w(x)$ to be nondecreasing in w for every $x < x'$. We assume without loss that every signal has positive ex ante probability; signals with zero ex ante probability are deleted from X.

For each $x \in X$, let $h(x)$ be the probability of observing signal x, let F_x be the posterior distribution after observing x, and let f_x be its density.[4]

We restrict attention to environments for which the status quo is optimal before information is acquired. We refer to this condition as zero asset under zero information (ZAZI):

$$\mathbf{0} \in \arg\max_{b \in B} \int u(w + b(w))\, \mathrm{d}F(w). \tag{ZAZI}$$

The ZAZI condition is a clean way to isolate the value of using information to make the asset choice: without information, the agent does not acquire any asset.[5]

3.1 Willingness to Pay for Information

An asset-selection *strategy* for the decision maker is a family $\beta = (\beta_x)_{x \in X} \in B^X$, describing the asset chosen after observing each signal realization. If the agent purchases information and uses strategy β, the CDF of her terminal wealth $G^\mu_\beta : \mathbb{R} \to [0, 1]$ is given by

$$G^\mu_\beta(m) = \sum_{x \in X} h(x) \int \mathbf{1}_{\{w + \beta_x(w) - \mu \leq m\}}\, \mathrm{d}F_x(w). \tag{1}$$

3. "TP2" stands for "totally positive of order 2". See Section 4 of Jewitt (1987) for a brief overview of the theory of total positivity.

4. Thus, $h(x) = \int h_w(x)\, \mathrm{d}F(w) > 0$ and $f_x(w) = \frac{h_w(x) f(w)}{h(x)}$.

5. Our hypotheses on F, u, and B guarantee that the maximization problems in (ZAZI) and (2) below admit solutions. See Appendix B for a proof.

A strategy is *optimal* for u if it maximises the expected payoff $\int u \, \mathrm{d}G^{\mu}_{\beta}$ across all strategies $\beta \in B^X$.[6] The agent (strictly) prefers to purchase information if

$$\max_{\beta \in B^X} \int u \, \mathrm{d}G^{\mu}_{\beta} \geq (>) \int u(w) \, \mathrm{d}F(w). \tag{2}$$

The following proposition shows that the agent's purchasing decision is summarized by a unique willingness to pay for information.

Proposition 1. *For any utility function $u \in U$, there exists a unique price $M(u) \in \mathbb{R}_+$ such that the agent strictly prefers not purchasing at prices $\mu > M(u)$, and she strictly prefers purchasing at prices $\mu < M(u)$.*

The proof of Proposition 1 is presented in Appendix B.

3.2 Comparative Statics in Risk Aversion

We are interested in establishing conditions on the economic environment, given by background risk and available assets such that more risk-averse agents are willing to pay more (resp. less) than less risk averse ones for information, prior to choosing an asset.

(I) We say that *willingness to pay for information decreases with risk aversion* if, for every $u, v \in U$ such that u is more risk averse than v and (ZAZI) holds for both,

$$M(u) \leq M(v).$$

(II) We say that *willingness to pay for information increases with risk aversion* if, for every $u, v \in U$ such that u is more risk averse than v and (ZAZI) holds for both,

$$M(u) \geq M(v).$$

The next section identifies sufficient conditions for the two orderings: investment menus under log-concave background risk yield decreasing willingness to pay, whereas insurance menus under downward-log-convex background risk yield increasing willingness to pay.

4 Main Comparative-Statics Results

This section establishes the paper's two positive comparative-statics results and develops their common distributional foundation. We first study investment menus under log-

6. The objective may equal $-\infty$ for all $\beta \in B^X$. In this case, all strategies are optimal. Note that (2) remains valid since the right-hand side is finite, by hypothesis.

concave background risk and then insurance menus under downward-log-convex background risk. The proof strategy has four steps: translate each comparative static into a single-crossing criterion, select a monotone optimal strategy, derive the conditional wealth comparison induced by that strategy, and aggregate the conditional comparisons over background risk. Finally, we present a new aggregation result, of interest in its own right, for single-crossing functions that provides the main technical ingredient of the analysis before completing the proofs of the main theorems.

4.1 Investment Menus under Log-Concave Background Risk

We first study environments in which the available assets increase exposure to background risk. Such assets are naturally interpreted as *procyclical*: they perform better in states in which background wealth is already high, and therefore amplify fluctuations in total wealth.

An *investment* is an asset whose payoff is nondecreasing in background wealth. Our results concern menus of investment opportunities rather than isolated assets. The following definition identifies menus whose elements can be ordered according to the amount of cyclical exposure they provide.

Definition 1. B is an *investment menu* if each $b \in B$ is an investment and there exists $w^* \in \mathbb{R} \cup \{\pm\infty\}$ and a complete ordering $\succeq$ of B such that, given any $b, b' \in B$ with $b' \succeq b$,

$$b'(w) \leq b(w) \qquad \text{for all } w \leq w^*,$$

and

$$b'(w) \geq b(w) \qquad \text{for all } w \geq w^*.$$

Thus, moving upward in the menu shifts payoffs from relatively poor states toward relatively rich states. Higher-ranked assets therefore provide greater exposure to aggregate conditions while preserving a common ordering across all states. Above the crossing point w^* higher-ranked assets pay more, while they pay less below it.

Investment menus arise naturally in several settings. One example is

$$B = \{\lambda b : \lambda \in L\},$$

where b is a nondecreasing F-integrable payoff, $L \subset \mathbb{R}_+$ is compact and contains 0, and there exists $w^* \in \mathbb{R} \cup \{\pm\infty\}$ such that $b(w) \leq 0$ for $w \leq w^*$ and $b(w) \geq 0$ for $w \geq w^*$. More generally, if $b^1, \ldots, b^n$ are investments with a common crossing point w^*, then

$$\{\mathbf{0}, b^1, b^1 + b^2, \ldots, b^1 + \cdots + b^n\}$$

is an investment menu, also with crossing point w^*. Additional examples are given in Section 6.2 and Example 3.

For investment menus, the relevant property of background risk is log concavity. Throughout this subsection we assume that the background-risk density is log concave, that is, $\{w \in \mathbb{R} : f(w) > 0\}$ is an interval on which $\log f$ is concave. Many standard distributions satisfy this condition, including the Gaussian distribution.

Our first main theorem shows that these two ingredients together imply a clean monotone comparative statics result.

Theorem 1. *If background risk is log concave and B is an investment menu, then willingness to pay for information is decreasing with risk aversion.*

The intuition begins with the benchmark of deterministic initial wealth studied by Cabrales, Gossner, and Serrano (2013, 2017). Investment opportunities are procyclical: they increase exposure precisely in states in which wealth is already high. Information is valuable because it allows the decision maker to take this exposure selectively. Less risk-averse decision makers are more willing to bear the resulting risk and therefore value information more highly.

Background risk adds a screening motive. Information allows the decision maker to avoid investment in catastrophic states while preserving investment opportunities in nearby, less adverse states. For a highly risk-averse agent, the value of information therefore depends on how effectively it separates these two parts of the lower tail. Purchasing information, however, also shifts wealth downward by a fixed amount in every state.

The thickness of tail distributions determines the balance between these effects. Under thin tails, as captured by log concavity, a fixed downward translation of wealth has a strong effect on lower-tail risk relative to the likelihood-ratio reweighting generated by the signal. The price of information therefore weighs heavily for agents who place greatest weight on the worst outcomes. Thus, under log concavity, the screening benefit does not overturn the usual investment logic: less risk-averse agents remain willing to pay more for information.

4.2 Insurance Menus under Downward-Log-Convex Background Risk

We now consider environments in which the available assets reduce exposure to background risk.

An *insurance* is an asset whose payoff is nonincreasing in background wealth and such that $w + b(w)$ is nondecreasing in w. Such assets transfer wealth from favorable states toward unfavorable ones, thereby reducing the cyclical component of wealth.

We extend this notion from individual assets to menus.

Definition 2. B is an *insurance menu* if each $b \in B$ is an insurance and there exists $w^* \in \mathbb{R} \cup \{\pm\infty\}$ and a complete ordering $\succeq$ of B such that, given any $b, b' \in B$ with $b' \succeq b$, $b'(w) \geq b(w)$ for all $w \leq w^*$ and $b'(w) \leq b(w)$ for all $w \geq w^*$.

One example of an insurance menu is

$$B = \{\lambda b : \lambda \in L\},$$

where $b : \mathbb{R} \to \mathbb{R}$ is nonincreasing, F-integrable, satisfies $\inf b < 0 < \sup b$, and $w + b(w)$ is nondecreasing, while L is a compact subset of $[0, 1]$ containing 0.

For insurance menus, the relevant property of background risk is a version of log convexity.

Definition 3. A density f is *downward log convex* if $S = \{w \in \mathbb{R} : f(w) > 0\}$ is convex and unbounded below, and $\log f$ is convex on S.[7]

Our second main result mirrors Theorem 1.

Theorem 2. *If background risk is downward log convex and B is an insurance menu, then willingness to pay for information is increasing with risk aversion.*

The economic force is the mirror image of the investment case. Insurance transfers wealth toward adverse realizations of background risk, and information is valuable because it allows this protection to be shifted toward the states in which it is most needed. For a highly risk-averse decision maker, the relevant benefit is therefore the improvement in the worst outcomes.

Purchasing information, however, also reduces wealth by a fixed amount in every state. The value of information depends on whether the gain from directing insurance toward catastrophic outcomes outweighs the deterioration in tail risk caused by this uniform wealth loss.

The thickness of the left tail of the distribution determines the balance between these effects. Under fat left tails, characterized by downward log convexity, fixed translations of wealth have a relatively weak effect on lower-tail probabilities, while likelihood-ratio updating can substantially change the relative weight placed on nearby adverse states. Information can therefore improve the targeting of insurance more than its price worsens downside risk. Since more risk-averse decision makers attach greater value to this reallocation of protection toward the worst states, willingness to pay for information increases with risk aversion.

The next subsection develops the common distributional logic behind the two results.

7. Note that, if f is downward log convex, then it has support $(-\infty, \omega]$ for some $\omega \in \mathbb{R}$. The requirement that the positive-density region be unbounded below is separate from log convexity. With support bounded below, sufficiently risk-averse agents may not acquire information.

4.3 A Distributional Approach

Although the economic mechanisms differ, both comparative-statics results are ultimately driven by the same distributional comparison. Purchasing information combines two opposing forces: a uniform downward wealth shift generated by its price, and a signal-contingent reallocation of wealth through asset choice. For investments, this reallocation screens exposure away from catastrophic states; for insurance, it shifts protection toward them. The proofs of Theorems 1 and 2 therefore proceed by comparing directly the distributions of terminal wealth with and without information.

The argument proceeds in four steps. First, Lemma 3 reduces each comparative static to a single-crossing comparison between terminal wealth with and without information. Second, Lemma 4 selects an optimal strategy that is monotone in the relevant menu order. Third, Lemmas 11 and 12, using Lemma 10, derive the conditional comparisons generated by investment and insurance menus. Fourth, Lemma 5 aggregates those comparisons over background risk by applying the state-dependent preservation result in Lemma 6. The auxiliary proofs are collected in Appendix C and Appendix E; Appendix D gives worked illustrations of the monotone strategies and terminal-wealth crossings.

Recall from Section 3 that G^{μ}_{β} is the terminal-wealth distribution after purchasing information at price $\mu \geq 0$ and implementing strategy $\beta \in B^X$. For a non-empty $I \subseteq \mathbb{R}$, a map $\psi : I \to \mathbb{R}$ is *single crossing* if, for all $m \leq m'$,

$$\psi(m) > 0 \quad \Longrightarrow \quad \psi(m') \geq 0.$$

Lemma 3 (Single-crossing criterion)**.** *Willingness to pay for information decreases with risk aversion if, for every $u \in U$ satisfying* (ZAZI) *and every $0 \leq \mu < M(u)$, there exists a strategy β that is optimal for u at price μ and such that $F - G^{\mu}_{\beta}$ is single crossing.*

Willingness to pay for information increases with risk aversion if, for every $u \in U$ satisfying (ZAZI) *and every $0 \leq \mu \leq M(u)$, there exists a strategy β that is optimal for u at price μ and such that $G^{\mu}_{\beta} - F$ is single crossing.*

Thus, the economic question is transformed into a geometric one: it is enough to show that $F - G^{\mu}_{\beta}$ in the decreasing case, or $G^{\mu}_{\beta} - F$ in the increasing case, is single crossing. Once the relevant CDF difference becomes strictly positive, it remains nonnegative at every higher cutoff. This lemma is a direct consequence of Theorem 1 in Jewitt (1989).

The second lemma shows that optimal strategies inherit the ordering structure of the asset menu. Say that a strategy $\beta \in B^X$ is *monotone* relative to $\succeq$ if $x \leq y$ implies $\beta_y \succeq \beta_x$.

Lemma 4 (Monotone optimal strategy)**.** *Suppose that $\succeq$ is a complete ordering of B such that $b' - b$ is single crossing whenever $b' \succeq b$. Then, for every $u \in U$ and every $\mu \geq 0$, there exists a strategy that is optimal for u at price μ and monotone relative to $\succeq$.*

For an investment menu, the lemma shows that higher signals lead to weakly higher-ranked investments. For an insurance menu, it shows that higher signals lead to weakly lower-ranked insurance positions.

It remains to understand how these monotone conditional decisions aggregate once background risk is integrated out.

In what follows, given random variables Y and W, we denote by F_Y the distribution function of Y, and by $F_{Y|W}$ a conditional distribution of Y given W.[8]

The first two lemmas reduce the proofs of the main theorems to a distributional comparison between wealth with and without information. The next lemma provides exactly this comparison. It shows that, under the appropriate tail condition on background risk, the single-crossing structure induced by monotone asset choices is preserved after adding background risk. This is the probabilistic step that connects the economic structure of investment and insurance menus to the distributional criterion of Lemma 3. Its proof is a direct application of the aggregation theorem established in the next subsection.

Lemma 5. *Let (Y^1, Y^2, W) be a random vector. Suppose that versions of $F_{Y^1|W}$ and $F_{Y^2|W}$ can be chosen such that there exist $\underline{w} \leq \bar{w}$ in $\mathbb{R} \cup \{\pm\infty\}$ for which, for every $y \in \mathbb{R}$, the map $w \mapsto F_{Y^1|W}(y|w-y) - F_{Y^2|W}(y|w-y)$ is nonpositive on $\{w \in \mathbb{R} : w \leq \underline{w}\}$, nondecreasing on $(\underline{w}, \bar{w})$, and nonnegative on $\{w \in \mathbb{R} : w \geq \bar{w}\}$.*

Then, the following two statements hold:

(I) If W admits a log-concave density and, for every $w \in \mathbb{R}$, $y \mapsto F_{Y^1|W}(y|w-y) - F_{Y^2|W}(y|w-y)$ is single crossing, then $F_{Y^1+W} - F_{Y^2+W}$ is single crossing.

(II) If W admits a downward-log-convex density with support $(-\infty, \omega]$, $y \mapsto F_{Y^2|W}(y|w-y) - F_{Y^1|W}(y|w-y)$ is single crossing for each $w \in \mathbb{R}$, and $F_{Y^1|W}(y|w-y) = F_{Y^2|W}(y|w-y)$ for all $y < w - \omega$ and $w < \bar{w}$, then $F_{Y^1+W} - F_{Y^2+W}$ is single crossing.

4.4 Aggregation under Background Risk

The final step is to aggregate the conditional wealth comparisons over background risk. Standard preservation results such as Karlin and Rubin (1956) do not apply directly because the conditional comparison depends on background wealth: both conditional signal probabilities and state-contingent asset payoffs vary with the state. The following result, proved in Appendix C, shows that single crossing is nevertheless preserved when

8. As usual, $F_{Y|W}$ denotes any measurable version of the conditional distribution satisfying

$$\int_E F_{Y|W}(y|w)\,\mathrm{d}F_W(w) = \Pr(Y \leq y,\ W \in E)$$

for every Borel set $E \subseteq \mathbb{R}$.

this state dependence is monotone and the aggregation kernel is TP2. It provides the common mathematical foundation for both comparative-statics theorems.

Lemma 6. *Let $I \subseteq \mathbb{R}$ be an interval, let $\phi : \mathbb{R} \times I \to \mathbb{R}, \kappa : \mathbb{R} \times I \to \mathbb{R}_+$, and suppose that κ is TP2. Assume that:*

(i) for every $m \in I$, the map $\ell \longmapsto \phi(\ell, m)$ is single crossing;

(ii) there exist $-\infty \leq \underline{m} \leq \bar{m} \leq \infty$ such that, for every $\ell \in \mathbb{R}$, the map $m \longmapsto \phi(\ell, m)$ is nonpositive on $I \cap (-\infty, \underline{m}]$, nondecreasing on $I \cap (\underline{m}, \bar{m})$, and nonnegative on $I \cap [\bar{m}, \infty)$;

(iii) for every ℓ and m in I, the map $k \longmapsto \phi(k, \ell)\kappa(k, m)$ is integrable.

Then $m \longmapsto \int_{\mathbb{R}} \phi(\ell, m)\kappa(\ell, m)\, d\ell$ is single crossing on I.

The novelty relative to the classical preservation result of Karlin and Rubin (1956) (Lemma 9 in Appendix C) is that the conditional comparison is allowed to vary with the aggregation variable. In our application, this variation reflects the interaction of the optimal signal-contingent strategy with background wealth: the conditional distribution of the selected payoff, and hence the conditional wealth comparison, varies with the state. The monotonicity condition in the second argument ensures that these comparisons move coherently with background wealth, while TP2 ensures that integration preserves their single-crossing structure. The classical result of Karlin and Rubin (1956) is recovered by taking $I = \mathbb{R}$ and ϕ independent of its second argument.

4.5 Proofs of the Main Theorems

We now combine the preceding results. Lemma 3 reduces each comparative-statics result to a single-crossing comparison between terminal wealth with and without information, while Lemma 4 provides an optimal monotone strategy. For investment menus, Lemma 5(I) aggregates the resulting conditional comparison under log-concave background risk, while Lemma 5(II) performs the same task for insurance menus under downward log convex background risk.

The remaining conditional properties of monotone strategies, necessary to apply Lemma 5, are established in Lemmas 11 and 12 in Appendix E.

Proof of Theorem 1. Let w^* and $\succeq$ satisfy Definition 1. If $w^* \in \{\pm\infty\}$, then either (ZAZI) fails for every nontrivial utility under consideration or every admissible asset is weakly dominated by the status quo, so the conclusion is immediate. Hence assume $w^* \in \mathbb{R}$.

Fix $u \in U$ and $\mu \geq 0$. Let χ denote the signal. By Lemma 4, applied to $\succeq$, there exists an optimal strategy β satisfying $\beta_y \succeq \beta_x$ whenever $x \leq y$, hence monotone relative to $\succeq$.

By Lemma 11, the conditional distributions associated with $Y^1 = 0$, $Y^2 = \beta_\chi(W) - \mu$ satisfy the hypotheses of Lemma 5(I). Hence $F - G^\mu_\beta$ is single crossing. The conclusion follows from Lemma 3. $\square$

Proof of Theorem 2. Let w^* and $\succeq$ satisfy Definition 2, and let f be downward log convex with support $(-\infty, \omega]$. If $w^* \in \{-\infty\} \cup [\omega, \infty]$, comparison with $\mathbf{0}$ makes every asset one-signed on the support of F. The ZAZI condition excludes any nonnegative nonzero asset, while every nonpositive asset is dominated by the status quo. The conclusion is therefore immediate. Hence assume $w^* \in (-\infty, \omega)$.

Let $\succeq^R$ be the reverse of $\succeq$: $b' \succeq^R b$ if and only if $b \succeq b'$. If $b' \succeq^R b$, then $b' - b$ is single crossing. Fix $u \in U$ and $\mu \geq 0$. By Lemma 4, applied to $\succeq^R$, there exists an optimal strategy β satisfying $\beta_y \succeq^R \beta_x$, equivalently $\beta_x \succeq \beta_y$, whenever $x \leq y$. Let χ denote the signal and define $Y^1 = \beta_\chi(W) - \mu$, $Y^2 = 0$. By Lemma 12, the associated conditional distributions satisfy the hypotheses of Lemma 5(II). Hence $G^\mu_\beta - F$ is single crossing. The conclusion follows from Lemma 3. $\square$

5 Comparative-Statics Reversals

Theorems 1 and 2 identify two broad classes of background-risk distributions under which the value of information is monotone in risk aversion. We now show that their tail restrictions have substantive force by constructing environments in which each comparative static reverses. For investment menus, the reversal arises under strict log convexity. For insurance menus, it arises under a super-exponential left tail, whose limiting behavior is opposite to downward log convexity. These constructions complement the positive results without asserting that their assumptions are necessary.

5.1 Investment Menus under Log-Convex Tails

We first consider investment menus under log-convex background risk. In contrast with Theorem 1, willingness to pay for information need no longer decrease with risk aversion. The following result is proved in Appendix F.2.

Proposition 2 (Investment Reversal under Log-Convex Tails)**.** *Suppose that f is continuously differentiable and strictly positive on $(-\infty, \omega)$ for some $\omega \in \mathbb{R}$, and that* $\log f$ *is strictly convex on this interval.*

Then there exist an investment menu $B = \{\mathbf{0}, b\}$, a binary MLR-ordered experiment α, and increasing, Lipschitz continuous, concave utility functions u, v satisfying (ZAZI) *such that u is more risk averse than v and*

$$M(u) > M(v).$$

The construction is based on an investment b that generates a large loss in an adverse wealth region and a gain otherwise. Without information, exposure to the loss makes the investment unattractive. The experiment α sometimes generates a positive signal in the favorable region, but never in the adverse region. The positive signal thus induces an investment decision.

Consider the calibrated price μ at which the benchmark decision maker v is indifferent between purchasing information and remaining uninformed, and let F and G denote the corresponding terminal-wealth distributions. In the extreme left tail, the favorable signal never occurs, so only the information price matters and $G > F$. The calibration yields $G < F$ on an intermediate lower region, whereas strict log convexity yields $G > F$ on a higher interval. Thus the CDF difference changes sign at least twice. The favorable intermediate region is calibrated to dominate the extreme-tail deterioration in the relevant integrated comparison. Greater risk aversion increases the relative weight placed on this net lower-region improvement.

5.2 Insurance Menus under Super-Exponential Left Tails

We next consider the opposite tail behavior to that implied by downward log convexity. We say that f has a *super-exponential left tail* if it is continuously differentiable and strictly positive on $(-\infty, \omega)$ for some $\omega \in \mathbb{R}$ and

$$\lim_{w \to -\infty} (\log f)'(w) = +\infty.$$

Under downward log convexity, $(\log f)'$ is nondecreasing and therefore remains bounded above as wealth tends to $-\infty$. Super-exponentiality imposes the opposite limiting behavior: the log slope becomes arbitrarily large in the extreme left tail. In particular, the left tail is thinner than that of any exponential distribution. The following proposition, proved in Appendix F.3, shows that under this tail behavior the insurance comparative statics of Theorem 2 may reverse.

Proposition 3 (Insurance Reversal under Super-Exponential Left Tails)**.** *Suppose that f has a super-exponential left tail. Then there exist an insurance menu $B = \{\mathbf{0}, b\}$, a binary MLR-ordered experiment α, and increasing, Lipschitz continuous, concave utility functions u, v satisfying* (ZAZI) *such that u is more risk averse than v and*

$$M(u) < M(v).$$

The benchmark decision maker v is risk neutral. The construction uses an insurance contract that pays a fixed amount in sufficiently adverse states and generates losses in more favorable ones. After a low signal, the contract pays the fixed amount throughout the posterior support and is therefore strictly preferred by every increasing decision maker.

Without information, the benchmark decision maker rejects the contract because its prior expected payoff is negative. She also rejects it after a high signal, which downweights the adverse states in which the contract pays. The information price is set equal to the expected gross payoff from this signal-contingent strategy, making v indifferent between purchasing information and remaining uninformed.

Let F denote the terminal-wealth distribution without information and G the terminal-wealth distribution when v purchases information and follows this strategy. In sufficiently adverse states, the low signal occurs with probability $q \in (0, 1)$, while a high signal leads to no insurance but still bears the information price. At a sufficiently low cutoff m, this high signal contributes $(1 - q)F(m + \mu)$ to $G(m)$.

Super-exponentiality implies that $F(m + \mu)/F(m) \to +\infty$ as $m \to -\infty$, so the price-only branch eventually makes $G(m) > F(m)$ despite the protection provided after the low signal. Since v is linear and indifferent, F and G have the same mean. The utility u is obtained by increasing the slope of v only in this extreme lower-tail region. This makes u more risk averse and gives greater weight to the deterioration there. By choosing the change sufficiently small, the prior and posterior insurance choices made by v remain optimal for u, while information becomes strictly unattractive at the benchmark price.

Together, Propositions 2 and 3 show that the tail restrictions in the positive theorems have substantive force: alternative tail geometries can reverse each comparative static, although the propositions do not establish necessity. Section 6.1 next isolates the local probability-mass comparison behind the positive and reversal results and separates it from the support requirement in the insurance theorem.

6 Extensions and Applications

This section first isolates the roles of tail curvature and support in the main theorems. It then applies the results to financial options and to continuous-state environments with multiple investment or insurance opportunities. The final subsection considers a related but distinct question: conditional on acquiring information, how does risk aversion affect the choice between an investment and an insurance opportunity?

6.1 The Role of the Tail Assumptions

The two-state examples in Section 2 were designed to isolate asset cyclicality from the geometry of background risk. The information is used either to take larger exposure selectively or to direct protection toward bad states. In those examples, no shape restriction on background risk is needed. In the general model, the tail assumptions become relevant because purchasing information and using it in the asset choice move wealth across the same cutoffs in opposite directions. We now show a simple local calculation that captures the geometry behind the more general aggregation arguments.

Fix a wealth cutoff m and two positive shifts a and μ. Consider the adjacent probability masses $A^+(m) = F(m+\mu) - F(m)$ and $A^-(m) = F(m) - F(m-a)$. Whenever $A^+(m) + A^-(m) > 0$, define their relative share by

$$P_{a,\mu}(m) = \frac{A^+(m)}{A^+(m) + A^-(m)} = \frac{F(m+\mu) - F(m)}{F(m+\mu) - F(m-a)}.$$

The numerator is the mass immediately above m that a uniform price μ can push below the cutoff. The term $A^-(m)$ is the mass immediately below m that an upward, state-contingent payoff of size a can pull above it. Thus $P_{a,\mu}(m)$ is a useful local measure of the strength of the price effect relative to the asset-reallocation effect. The full proofs also have to account for endogenous asset choice and the signal structure, but this adjacent-mass comparison is the tail geometry that makes the aggregation work.

The curvature of $\log f$ determines how this balance varies with the cutoff whenever the two adjacent intervals lie in the positive-density region, under the hypotheses of Lemma 13. Under strict log concavity, $P_{a,\mu}(m)$ decreases with m, so the price effect becomes relatively more important as the cutoff moves farther into the left tail. Under strict log convexity, $P_{a,\mu}(m)$ increases with m, so the asset-reallocation effect becomes relatively more important farther into the left tail.

These are the local geometries behind the investment and insurance results, respectively.

These monotonicity statements are formalized in Lemma 13. The two reversal results use different tail comparisons. Strict log convexity reverses the adjacent-mass ordering behind the investment result. The insurance reversal instead exploits super-exponentiality: for every $\mu > 0$, $F(m+\mu)/F(m) \to +\infty$ as $m \to -\infty$, and consequently $P_{a,\mu}(m) \to 1$. Thus a branch that bears the information price without receiving insurance eventually dominates the baseline lower-tail probability. A log-linear density, such as $f(w) = e^w$ on $(-\infty, 0]$, lies at the boundary of the adjacent-mass comparison because the corresponding ratio is constant.

In addition, the support assumption in the insurance theorem does different work from log convexity. Curvature controls the relative mass of neighboring regions away from a boundary. By requiring the support to be unbounded below, we ensure that this comparison remains meaningful arbitrarily far into the bad tail. With a finite lower endpoint, sufficiently risk-averse agents can become dominated by behavior near that hard boundary and may cease to purchase costly information, so an increasing comparative statics result cannot be guaranteed in general. This is why our definition of downward log convexity combines convexity of $\log f$ with support extending to $-\infty$. The reflected Lomax distribution used in Example 4 satisfies both requirements.

The assumptions therefore have distinct roles. The investment-versus-insurance menu condition determines the direction in which information reallocates wealth across states. The tail curvature determines how that reallocation compares with the universal wealth loss from paying for information. And, on the insurance side, the support condition rules out a lower-bound effect that would otherwise interfere with the tail comparison.

The next two subsections show how these ingredients appear in economically familiar environments.

6.2 Financial Options

The sufficient conditions in Theorems 1 and 2 apply to option payoffs when the underlying payoff is a monotone function of background wealth. Let $S = s(w)$, where s is nondecreasing, and let $\pi \geq 0$. A European call with strike K and premium π has net payoff $b_C(w) = (s(w) - K)^+ - \pi$. Assume that b_C is F-integrable and has a zero w^*. Then, for any compact $L \subseteq \mathbb{R}_+$ containing 0, $B_C = \{\lambda b_C : \lambda \in L\}$ is an investment menu: if $\lambda' \geq \lambda$, then $\lambda' b_C \leq \lambda b_C$ below w^* and $\lambda' b_C \geq \lambda b_C$ above w^*. Under log concavity of the background-risk density, Theorem 1 implies that, among decision makers satisfying ZAZI, willingness to pay for information before choosing the call position decreases with risk aversion.

Put options provide the corresponding insurance example. Consider a put written directly on wealth, with strike K, premium $\pi \geq 0$, and net payoff $b_P(w) = (K - w)^+ - \pi$. This payoff is nonincreasing and is F-integrable under the maintained first-moment assumption. For every $\lambda \in [0, 1]$, the map $w + \lambda b_P(w)$ is nondecreasing: its slope is $1 - \lambda$ below K and 1 above K. Since b_P crosses zero at $w^* = K - \pi$, $B_P = \{\lambda b_P : \lambda \in [0, 1]\}$ is an insurance menu. Under downward log convexity of the background-risk density, Theorem 2 implies that, among decision makers satisfying ZAZI, willingness to pay for information before choosing the put position increases with risk aversion.

These examples also illustrate why the menu condition is substantive. A single call with variable position size satisfies the investment-menu condition, and a single put with limited position size satisfies the insurance-menu condition. By contrast, an arbitrary collection of options with different strikes and premia need not satisfy either condition, because different options may cross the status quo at different wealth levels. The theorems therefore apply most directly to derivative menus that can be ordered by a common exposure parameter.

6.3 Continuous-State Examples with Multiple Assets

We now provide continuous-state analogues of the electric-car and prefabricated-house examples in Section 2. The diagnostic in Section 6.1 explained why distributional shape matters; the purpose of the present examples is complementary. We want to show that the menu, tail-shape, and support assumptions of the main theorems can arise together in economically natural environments with several assets. The state variable w should be interpreted as the realization of the general economic environment, or equivalently as the businessperson's background wealth. Higher values of w correspond to stronger aggregate conditions.

Let the information structure be binary, $X = \{L, H\}$. For some $\gamma > 0$ and threshold w^*, suppose

$$h_w(H) = \frac{\exp(\gamma(w - w^*))}{1 + \exp(\gamma(w - w^*))}, \quad h_w(L) = \frac{1}{1 + \exp(\gamma(w - w^*))}.$$

Then $h_w(H)/h_w(L) = \exp(\gamma(w - w^*))$, which is increasing in w. Hence the experiment is MLR-ordered. The signal H is good news about the aggregate state, while L is bad news.

Example 3 (A portfolio of electric-car investments). Suppose that a businessperson with CARA preferences, $u_r(m) = -\exp(-rm)$, $r > 0$, is deciding how much to invest in a collection of complementary projects related to electric mobility: for instance, an electric-car platform, a battery facility, charging infrastructure, and software. These projects are cyclical: they perform well when the aggregate economy is strong and perform poorly when the aggregate economy is weak.

Assume that background wealth W has a log-concave density. For example, let $W \sim N(m, \sigma^2)$. For each project $i = 1, \ldots, n$, let $e_i : \mathbb{R} \to \mathbb{R}$ denote the net payoff from project i.

Assume that each e_i is nondecreasing, F-integrable, and has the same crossing point w^*: $e_i(w) \leq 0$ for $w \leq w^*$, $e_i(w) \geq 0 \quad$ for $\;w \geq w^*$. A simple parametric specification is $e_i(w) = a_i(w - w^*)$, $a_i > 0$. Let the menu consist of cumulative investment plans, $b_E^k(w) = \sum_{i=1}^k e_i(w), \qquad k = 1, \ldots, n$, together with the status quo: $B_E = \{\mathbf{0}, b_E^1, \ldots, b_E^n\}$. Thus b_E^1 corresponds to investing only in the core electric-car platform, b_E^2 adds a second complementary project, and so on.

By the cumulative-menu construction following Definition 1, the monotonicity and common-crossing assumptions on the e_i imply that B_E is an investment menu, ordered by $b_E^{k'} \succeq b_E^k$ if and only if $k' \geq k$.

In the Gaussian linear specification, with $W \sim N(m, \sigma^2)$ and $e_i(w) = a_i(w - w^*)$, write $A_k = \sum_{i=1}^k a_i$. Then $w + b_E^k(w) = (1 + A_k)w - A_k w^*$. If $w^* \geq m$, the status quo is optimal under the prior for every CARA coefficient $r > 0$. Indeed,

$$\mathbb{E}\big[\exp -r(W + b_E^k(W))\big] = \exp\bigg(-r\big((1 + A_k)m - A_k w^*\big) + \frac{r^2}{2}(1 + A_k)^2\sigma^2\bigg),$$

which is increasing in $A_k \geq 0$ whenever $w^* \geq m$. Since CARA utility is the negative of this expression, expected utility is maximized at $A_k = 0$, i.e. at the status quo.

The conclusion of Theorem 1 applies to this example. Thus, if $r' > r > 0$, then

$M(u_{r'}) \leq M(u_r)$.

Example 4 (A portfolio of prefabricated-house investments). Consider now a businessperson who can invest in several lines of cheap prefabricated housing. These houses are most profitable in bad aggregate states, when households substitute toward inexpensive housing, and are less profitable in good aggregate states, when households demand higher-quality housing. Thus, these projects act like insurance against poor aggregate conditions.

Assume that background wealth W has a downward-log-convex density with the support required by Theorem 2. A convenient example is a reflected Lomax (Pareto Type II) distribution. Let Y have density

$$g(y) = \frac{\eta c^\eta}{(c+y)^{\eta+1}}, \qquad y \geq 0,$$

where $c > 0$ and $\eta > 1$, and set $W = \omega - Y$. Then W has support $(-\infty, \omega]$ and density

$$f(w) = \frac{\eta c^\eta}{(c+\omega-w)^{\eta+1}}, \qquad w \leq \omega.$$

Moreover,

$$\log f(w) = \log(\eta c^\eta) - (\eta+1)\log(c+\omega-w),$$

so

$$\frac{\mathrm{d}^2}{\mathrm{d}w^2}\log f(w) = \frac{\eta+1}{(c+\omega-w)^2} > 0$$

on $(-\infty, \omega)$. Hence f is downward log convex. The restriction $\eta > 1$ also gives a finite first moment, as required in the general model. This distribution preserves the intended fat-left-tail interpretation of the insurance example while satisfying the support assumptions of the theorem.

Let $p_i : \mathbb{R} \to \mathbb{R}$ denote the net payoff from the (i)-th prefabricated-house project. Assume that each p_i is nonincreasing, F-integrable, and has the same crossing point $w^* < \omega$: $p_i(w) \geq 0 \quad \text{for} \quad w \leq w^*$, $p_i(w) \leq 0 \quad \text{for } w \geq w^*$. A simple parametric specification is $p_i(w) = a_i(w^* - w)$, $a_i > 0$. Let $b_P^k(w) = \sum_{i=1}^k p_i(w)$, $k = 1, \ldots, n$, and define $B_P = \{\mathbf{0}, b_P^1, \ldots, b_P^n\}$. Assume in addition that $w + b_P^k(w)$ is nondecreasing for every k. Thus b_P^1 corresponds to investing in one prefabricated-house line, b_P^2 adds a second line, and so on.

Each b_P^k is nonincreasing, and $w + b_P^k(w)$ is nondecreasing by assumption. In the linear specification, the latter property follows from $A_n := \sum_{i=1}^n a_i \leq 1$, since $A_k \leq A_n$ and $w + b_P^k(w) = (1 - A_k)w + A_k w^*$. Finally, the common-crossing assumption implies that, for $k' \geq k$, $b_P^{k'} - b_P^k$ is nonnegative below w^* and nonpositive above it. Hence B_P is an insurance menu under the order $b_P^{k'} \succeq b_P^k$ if and only if $k' \geq k$.

To verify ZAZI for a nontrivial ordered family, specialize to $w^* = 0$ and assume

$\omega > c/(\eta - 1)$, so $\mathbb{E}[W] > 0$. For $u_q(m) = m - q(-m)^+$, define $\bar{q} = \mathbb{E}[W]/\mathbb{E}[(-W)^+] > 0$. Each u_q is increasing, concave, and Lipschitz, and $u_{q'}$ is more risk averse than u_q whenever $q' \geq q$. Since $W + b_P^k(W) = (1 - A_k)W$ and u_q is positively homogeneous,

$$\mathbb{E}[u_q(W + b_P^k(W))] = (1 - A_k)\mathbb{E}[u_q(W)] \leq \mathbb{E}[u_q(W)]$$

for every $q \in [0, \bar{q}]$. Hence ZAZI holds throughout this family. By Theorem 2, if $0 \leq q \leq q' \leq \bar{q}$, then $M(u_{q'}) \geq M(u_q)$.

6.4 Mixed Asset Menus

The preceding applications concern menus ordered entirely as investments or entirely as insurance. A mixed menu raises a distinct question. The following example does not compare willingness to pay for information; conditional on purchasing full information, it shows how risk aversion changes whether the decision maker selects an investment or an insurance opportunity.

Example 5. Suppose that a businessperson with CARA utility $u_r(m) = -e^{-rm}$, $r > 0$, is deciding between the two assets described in Examples 1 and 2 in Section 2. As in those examples, there are two states of the world: high, and low. The payoff from not opting for any asset is (k_H, k_L), with $k_H > k_L$.

The net payoff from undertaking the electric-car project is $H_E > 0$ in state H and $-L_E < 0$ in state L. The net payoff from undertaking the prefabricated-house project is $-H_P < 0$ in state H and $L_P > 0$ in state L, with $k_L + L_P < k_H - H_P$. Because she can oversee only one company, she selects one project before observing a fully revealing signal. After observing the state, she may undertake the selected project or decline it at zero payoff.

Assume that the prior probability of the state being low, denoted by p, is such that:

$$(1 - p)H_E < pL_E \qquad \text{and} \qquad pL_P < (1 - p)H_P.$$

These inequalities make both projects unattractive to a risk-neutral decision maker without information. They do not imply ZAZI for every $r > 0$: there is a unique $\bar{r}_P > 0$ such that the status quo is optimal relative to the full uninformed menu exactly when $0 < r \leq \bar{r}_P$. The assumptions below do not order $\bar{r}_P$ and the project-selection threshold r_0, so the ranking result is conditional on purchasing information and does not by itself establish that both project types are selected within the ZAZI domain.

Conditional on purchasing information, the price I multiplies the exponential losses from E and P by the common factor e^{rI}. Hence E is weakly preferred to P if and only if $\rho(r) \geq 1$, where ρ is defined below.

Lemma 7. Let $0 < p < 1$, $H_E, L_P > 0$, and $k_H - k_L > L_P$. Define, for $r > 0$,

$$\rho(r) := \frac{1-p}{p} e^{-(k_H - k_L)r} \frac{1 - e^{-H_E r}}{1 - e^{-L_P r}}.$$

Then ρ is strictly decreasing. If

$$\frac{1-p}{p}\frac{H_E}{L_P} > 1, \tag{3}$$

then $\rho(r) = 1$ has a unique solution $r_0 > 0$, with $\rho(r) > 1$ for $0 < r < r_0$ and $\rho(r) < 1$ for $r > r_0$. If inequality (3) *is reversed weakly, then $\rho(r) < 1$ for every $r > 0$.*

After full revelation, project E yields H_E in state H and zero in state L, while project P yields zero in state H and L_P in state L. Thus (3) says that a risk-neutral decision maker strictly prefers E to P. Conditional on purchasing information, agents with $0 < r < r_0$ choose E, agents with $r > r_0$ choose P, and the agent with $r = r_0$ is indifferent. If the inequality in (3) is reversed weakly, every agent with $r > 0$ chooses P over E.

7 Related Literature

Our analysis is related to four strands of literature: the classical theory of risk and comparative risk aversion, work on risky investment and insurance decisions, the literature on information acquisition and the value of information, and the macroeconomics and finance literature emphasizing tail risk.

Risk, risk aversion, and risky decisions. The preference comparison used in this paper evolves from the classical expected-utility theory of risk aversion. Pratt (1964) and Arrow (1971) develop the local and global notions of risk aversion that underlie comparisons of certainty equivalents and willingness to pay for insurance. Rothschild and Stiglitz (1970, 1971) provide the canonical distributional notion of an increase in risk, while Diamond and Stiglitz (1974) study how increases in risk interact with increases in risk aversion. These papers are directly relevant to ours because our proofs compare terminal-wealth distributions and ask how those comparisons are ranked as preferences become more risk averse. Because initial wealth is random in our model, Kihlstrom, Romer, and Williams (1981) provides a useful benchmark: it identifies conditions under which Arrow–Pratt comparisons of risk premia survive independent random initial wealth. Our setting allows asset payoffs to depend on the background-wealth state and compares information prices rather than risk premia. Related work develops stronger comparative notions of risk aversion and the effects of background risk. See, among others, Ross (1981), Jewitt (1987, 1989), and Gollier and Pratt (1996) and the synthesis in Gollier (2001).

Recent work provides behavioral foundations for risk aversion based directly on insurance choices. Maccheroni et al. (2025) show that weak risk aversion is equivalent to propensity to full insurance, while strong risk aversion is equivalent to propensity to several forms of partial insurance and hedging. They obtain parallel comparative results: Yaari's ordering corresponds to comparative propensity to full insurance, whereas Ross's stronger ordering corresponds to comparative propensity to partial insurance. Côté, Wang, and Wu (2025) develop this connection further through the notion of risk–insurance parity. They characterize the classes of indemnity functions associated with weak and strong risk aversion and introduce intermediate attitudes characterized by propensity to deductible-only and limit-only contracts.

These characterizations concern comparisons between an insurance contract and an equidistributed alternative whose dependence with the background risk differs. Our comparison concerns a different object: willingness to pay for an experiment that permits an endogenous, signal-contingent choice from an asset menu. Consequently, terminal wealth with and without information need not have the same marginal distribution. Moreover, the comparative characterization of partial-insurance propensity in Maccheroni et al. (2025) uses Ross's strong ordering, whereas our insurance result uses the standard concave-transformation ordering under restrictions on the experiment, the asset menu, and the tail geometry of background risk.

A related literature studies optimal insurance in the presence of background risk. Chi and Wei (2020) characterize optimal indemnities under general dependence between insurable and background risks and show that the resulting contracts need not have the classical piecewise-linear form. Hinck and Steinorth (2023) consider loss-dependent background risk and identify conditions involving risk vulnerability, prudence, and higher-order risk attitudes under which background risk raises insurance demand. Our background risk also affects the value of insurance, but it is itself the object about which information is acquired. The agent purchases a signal about background wealth and subsequently selects an asset, so the comparative statics is governed by the interaction among information, endogenous asset choice, and the distributional geometry of background wealth.

Recent work also studies insurance menus and insurance demand outside the standard expected-utility environment. Under dual utility and adverse selection, Gershkov et al. (2023) derive optimal menus consisting of deductible–premium or coverage-limit–premium pairs. Jaspersen, Peter, and Ragin (2023) analyze how probability weighting affects coinsurance, deductible choice, and insurance against low-probability losses, identifying conditions under which probability weighting and utility curvature act as substitutes. More broadly, Rieger-Fels (2024) emphasizes that insurance demand may arise from state-dependent financial needs rather than risk aversion alone. We maintain expected utility in order to isolate how comparative risk aversion affects information acquisition when information can be used either to expand procyclical exposure or to target insurance toward adverse states.

Risk aversion also plays a key role in investment and production decisions under uncertainty. For example, Sandmo (1971) studies the behavior of a risk-averse competitive firm under price uncertainty, while portfolio-choice work such as Samuelson (1969) analyzes how risky investment responds to preferences and investment opportunities. Our distinction between investment and insurance menus is in the same spirit, but our comparative static concerns the value of acquiring information before the asset choice. In addition, our equilibrium motivation is related to the classic information-market analysis of Grossman and Stiglitz (1980). When acquiring information is costly, risk preferences can affect which agents become informed and therefore, in richer market models, who bears risk and how information is incorporated into prices. The insurance side is similarly connected to the role of information in insurance markets emphasized by Rothschild and Stiglitz (1976).

Risk aversion and the value of information. A separate literature asks directly how the value or demand for information changes with risk and risk aversion. Gould (1974) studies how changes in payoff risk affect information value, while Hilton (1981) shows that attributes of the decision problem and decision maker need not have a uniform directional effect. Freixas and Kihlstrom (1984) obtains decreasing information demand under specific assumptions, and Willinger (1989) identifies a local CARA condition under which the sign depends on how information changes the risk of the optimal action. Alepuz and Urbano (1995) separates the ex post benefit of reduced uncertainty from the ex ante riskiness of returns to information, while Eeckhoudt and Godfroid (2000) illustrates why the two effects need not deliver a uniform sign. Nadiminti, Mukhopadhyay, and Kriebel (1996) derives conditions for either sign and emphasizes that the method of payment for information matters. Kihlstrom (2022) surveys decreasing investment-side results in Cabrales, Gossner, and Serrano (2013, 2017) and the CARA portfolio analysis of Losq and Sobti.

Our earlier work, Cabrales, Gossner, and Serrano (2013, 2017), constructs complete orderings of information based on investors' willingness to pay and obtains an inverse relation between the value of information and risk aversion in an environment with a safe outside option. The present paper explains why that decreasing relation is natural for investment menus but need not extend to insurance-like opportunities. More generally, our contribution is to replace a collection of apparently conflicting examples with conditions stated in terms of two economically interpretable primitives. We focus on asset protection or expansion of risk, and the tail geometry of background risk.

Tails in macroeconomics and finance. The emphasis on tail geometry connects the paper to a large macro-finance literature in which low-probability, severe outcomes have first-order effects on quantities and asset prices. Rietz (1988) showed that rare market crashes can substantially alter equilibrium risk premia; Barro (2006) revived and quantified the rare-disaster mechanism using twentieth-century macroeconomic disasters and showed that other puzzles could be solved using the same mechanism; and Gabaix (2012) showed how time-varying disaster severity can account for a broad set of macro-finance facts. In business-cycle analysis, Gourio (2012) shows that variation in disaster risk can move output, employment, investment, and asset prices. A complementary literature studies the endogenous origins of aggregate tail events: Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), for example, show how sectoral heterogeneity and input-output linkages can generate macroeconomic tail risks even when ordinary fluctuations away from the tails are close to Gaussian.

Our use of tails is different but complementary. Rather than introducing disaster states to explain aggregate moments or asset prices, we impose shape restrictions on the distribution of background wealth and ask how those restrictions govern comparative statics of information demand. Log concavity captures a relatively thin-tail geometry under which a fixed downward wealth translation has a strong lower-tail effect, while downward log convexity captures a fat-left-tail geometry under which the targeting benefit of insurance can dominate that translation. Thus, tail behavior enters here as a primitive condition that determines how information, asset payoffs, and risk aversion interact.

Contribution relative to the literature. Taken together, these strands clarify the paper's contribution. The classical risk literature provides the ordering of preferences and distributions. The investment and insurance literature provides the economic interpretation of risk-increasing and risk-protecting assets. The information literature documents that the comparative static of information value with respect to risk aversion can have either sign. Finally, the macro-finance literature shows why the tails of economic risk can be quantitatively and qualitatively decisive. Our results connect these ideas by showing that asset cyclicality and tail geometry jointly select the sign of the information-acquisition comparative static in a single general framework.

8 Conclusion

We study how comparative risk aversion changes willingness to pay for information acquired before an asset choice under background risk. For finite MLR-ordered experiments and decision makers satisfying ZAZI, willingness to pay decreases for investment menus under log-concave background risk and increases for insurance menus under downward-log-convex background risk. The common mechanism is the interaction between the uniform wealth loss from the information price and the signal-contingent reallocation generated by asset choice; our state-dependent single-crossing result provides the aggregation step linking this mechanism to the two comparative statics. The investment and insurance reversals show that the tail restrictions have substantive force without providing a necessity characterization. These results isolate a primitive determinant of information acquisition that can be incorporated into market models, whose equilibrium analysis remains outside the scope of the paper.

Our framework assumes that asset payoffs are deterministic functions of background wealth. A natural extension would allow their conditional distributions to vary with background wealth, thereby accommodating more general dependence between asset returns and background risk. This extension lies beyond the scope of the paper, and we hope to contribute to this question in future research.

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Appendices

A Comparative Statics Under Full Revelation

We now spell out the logic of the comparative statics when the signal is fully revealing.

Let W be an integrable random variable with distribution F, and let B be a finite set of integrable asset payoffs $b : \mathbb{R} \to \mathbb{R}$ containing the status quo random variable $\mathbf{0}$ which equals 0 a.s. The decision maker has a strictly increasing utility function $u \in U$. We restrict attention to utilities for which the status quo is optimal without information: $\mathbf{0} \in \arg\max_{b \in B} \mathbb{E}[u(W + b(W))]$. If she pays $\mu \geq 0$ to observe W before choosing an asset, monotonicity of u implies that she chooses an asset with maximal payoff at the realized state. Let $c(w) = \max_{b \in B} b(w)$. Her terminal wealth with complete information is therefore $T_\mu(W) = W + c(W) - \mu$. We define her willingness to pay for complete information by:

$$M^{FI}(u) = \sup\{\mu \geq 0 : \mathbb{E}[u(T_\mu(W))] \geq \mathbb{E}[u(W)]\}.$$

An asset is an *investment* if b is nondecreasing. An asset is an *insurance* if b is nonincreasing and $w + b(w)$ is nondecreasing.

Proposition 4. *Under complete information:*

(i) if every asset in B is an investment, willingness to pay for information decreases with risk aversion;

(ii) if every asset in B is an insurance, willingness to pay for information increases with risk aversion.

These conclusions hold for any distribution of background wealth.

Proof. Suppose first that every asset is an investment. Then c is nondecreasing, so T_μ is increasing and $T_\mu(w) - w = c(w) - \mu$ is nondecreasing. Consequently, the distributions of $T_\mu(W)$ and W have the following *single-crossing property*: once the CDF of W lies above that of $T_\mu(W)$, it remains above it at all higher wealth levels. By the standard single-crossing characterization of comparative risk aversion of Jewitt (1987), if a more risk-averse decision maker prefers $T_\mu(W)$ to W, then every less risk-averse decision maker does as well. Hence $M^{FI}(u)$ decreases with risk aversion.

Suppose instead that every asset is an insurance. Then c is nonincreasing, while $w + c(w) = \max_{b \in B}\{w + b(w)\}$ is nondecreasing. Thus T_μ is increasing and $T_\mu(w) - w$ is nonincreasing. The preceding single-crossing comparison is reversed: once the CDF of $T_\mu(W)$ lies above that of W, it remains above it at all higher wealth levels. Hence, if a less risk-averse decision maker prefers $T_\mu(W)$ to W, every more risk-averse decision maker does as well. Therefore $M^{FI}(u)$ increases with risk aversion. $\square$

Complete information eliminates the difficulty created by background risk in the general model. For investments, the gain from adapting the asset choice is increasing in background wealth; for insurance, it is decreasing; this is enough to ensure single crossing. With imperfect information, asset choice instead depends on a noisy signal, and the resulting conditional comparisons must be aggregated across background-wealth states. In that case, further distributional assumptions are needed to ensure aggregation preserves comparative statics.

B Proof of Proposition 1

We begin by establishing existence of optimal strategies.

Lemma 8. *Let $u \in U$. The integral $\int u(w + b(w))\,\mathrm{d}F(w)$ exists and lies in $\mathbb{R} \cup \{-\infty\}$ for all $b \in B$, and the maximization problem*

$$\max_{b\in B} \int u(w + b(w))\,\mathrm{d}F(w)$$

admits a solution. Given $x \in X$, the integral $\int u(w + b(w) - \mu) f_x(w)\,\mathrm{d}w$ exists and lies in $\mathbb{R} \cup \{-\infty\}$ for all $b \in B$, and the maximization problem

$$\max_{b\in B} \int u(w + b(w) - \mu) f_x(w)\,\mathrm{d}w$$

admits a solution.

Proof of Lemma 8. For each $x \in X$, let F_x be the CDF of f_x. Fix $F' \in \{F\} \cup \{F_x : x \in X\}$ and $\mu \geq 0$. It suffices to show that, given any $b \in B$, the integral $\int u(w+b(w)-\mu)\,\mathrm{d}F'(w)$ exists and lies in $\mathbb{R} \cup \{-\infty\}$, and that the map $b \mapsto \int u(w + b(w) - \mu)\,\mathrm{d}F'(w)$ is upper-semicontinuous with respect to the $L^1(F)$ norm, as this guarantees that the maxmisation problem

$$\max_{b\in B} \int u(w + b(w) - \mu)\,\mathrm{d}F'(w)$$

admits a solution, since B is compact.

Any F-integrable map is also F'-integrable and convergence in $L^1(F)$ implies convergence in $L^1(F')$ since, given any $x \in X$, $f_x(w)\,\mathrm{d}w = h_w(x)\,\mathrm{d}F(w)/h(x)$ and $w \mapsto h_w(x)$ is bounded.

Choose $\delta > \limsup_{m\to\infty} u(m)/m$ and $m' > 0$ such that $u(m)/m \leq \delta$ for all $m \geq m'$. Since u is increasing, u lies pointwise weakly below the map $\bar{u}(m) = \max\{u(m'), \delta m\}$. Given $b \in B$, the map $w \mapsto \bar{u}(w + b(w))$ is F-integrable since b and the identity function are; hence it is F'-integrable. Then, the integral $\int u(w+b(w)-\mu)\,\mathrm{d}F'(w)$ exists and lies in $\mathbb{R}\cup\{-\infty\}$, since $u(m-\mu)\leq \bar{u}(m)$ for all $m\in\mathbb{R}$.

To establish upper-semicontinuity, suppose that $b_n\to b$ in $L^1(F)$ and that $\int u(w+b_n(w)-\mu)\,\mathrm{d}F'(w)$ converges. Since $\bar{u}$ is Lipschitz, $\bar{u}(w+b_n(w))\to\bar{u}(w+b(w))$ in $L^1(F')$, and their integrals converge. Moreover, after passing to a subsequence if necessary, we may assume without loss that $b_n\to b$ F'-a.e. Let $v:\mathbb{R}\to\mathbb{R}$ be given by $v(m)=\bar{u}(m)-u(m-\mu)$. v is continuous, so that $v(w+b_n(w))\to v(w+b(w))$ F'-a.e. Then

$$\begin{aligned}
&\lim_{n\to\infty}\int u(w+b_n(w)-\mu)\,\mathrm{d}F'(w)\\
&\leq \lim_{n\to\infty}\int \bar{u}(w+b_n(w))\,\mathrm{d}F'(w)-\liminf_{n\to\infty}\int v(w+b_n(w))\,\mathrm{d}F'(w)\\
&\leq \int \bar{u}(w+b(w))\,\mathrm{d}F'(w)-\int \lim_{n\to\infty} v(w+b_n(w))\,\mathrm{d}F'(w)\\
&= \int u(w+b(w)-\mu)\,\mathrm{d}F'(w)
\end{aligned}$$

where the inequality follows from Fatou's lemma, since v is positive. □

Proof of Proposition 1. For all $u\in U$ and $\mu\geq 0$,

$$V(u,\mu)=\max_{\beta\in B^X}\int u\,\mathrm{d}G^{\mu}_{\beta}$$

is well defined by Lemma 8. Fix $u\in U$. Since u is increasing and F-integrable, $\int u\,\mathrm{d}F>\lim_{m\to-\infty}u(m)$. Then, it suffices to show that $V(u,\mu)<V(u,\mu')$ for all $0\leq\mu'<\mu$ such that $V(u,\mu)>-\infty$, that $V(u,0)\geq\int u\,\mathrm{d}F$ and that $\lim_{\mu\to\infty}V(u,\mu)\leq\lim_{m\to-\infty}u(m)$. Indeed, this guarantees that $M(u)=\sup\{\mu\in\mathbb{R}:V(u,\mu)\geq\int u\,\mathrm{d}F\}$ is well defined and lies in $\mathbb{R}$, that $V(u,\mu)>\int u\,\mathrm{d}F$ for all $0\leq\mu<M(u)$, and $V(u,\mu)<\int u\,\mathrm{d}F$ for all $\mu>M(u)$.

Fix $0\leq\mu'<\mu$ such that $V(u,\mu)>-\infty$, and let β achieve the maximum at μ. Since X is finite, $V(u,\mu)\in\mathbb{R}$ by Lemma 8. Then,

$$V(u,\mu)=\int u\,\mathrm{d}G^{\mu}_{\beta}<\int u\,\mathrm{d}G^{\mu'}_{\beta}\leq V(u,\mu'),$$

where the strict inequality holds since u is increasing.

Since $\mathbf{0}\in B$, $V(u,0)\geq\int u\,\mathrm{d}F$. To prove that $\lim_{\mu\to\infty}V(u,\mu)=\lim_{m\to-\infty}u(m)$, fix a sequence $\mu_n\to\infty$ and choose β^n optimal at μ_n. Set $E^n_x=\{w:w+\beta^n_x(w)>\mu_n/2\}$ for all $x\in X$ and $n\in\mathbb{N}$, and $E^n=\bigcup_{x\in X}E^n_x$. Let $c=\sup\big\{\int|w+\beta^n_x(w)|\,\mathrm{d}F(w):n\in\mathbb{N},\ x\in X\big\}$ and ν_F be the probability measure induced with F. Since $\int|w|\,\mathrm{d}F<\infty$ and B is compact in $L^1(F)$, the family of maps $(w\mapsto w+b(w):b\in B)$ is uniformly integrable; that is, $c<\infty$ and for any sequence of measurable sets $(D^n)_{n\in\mathbb{N}}$ such that $\nu_F(D^n)\to 0$, $\sup_{b\in B}\int_{D^n}|w+b(w)|\,\mathrm{d}F(w)\to 0$. Since $c<\infty$, Markov's inequality yields

$\nu_F(E^n) \le 2|X|c/\mu_n \to 0$. Choose $\delta > 0$ and $C \ge 0$ such that $u(m) \le C + \delta \max\{0, m\}$ for every $m \in \mathbb{R}$. Then

$$V(u, \mu_n) \le u(-\mu_n/2)(1 - \nu_F(E^n)) + C\nu_F(E^n) + \delta \sum_{x\in X} \int_{E^n} \max\{0, w + \beta_x^n(w)\} \,\mathrm{d}F(w).$$

The second and third terms vanish as $n \to \infty$ (the latter by uniform integrability), while the first tends to $\lim_{m\to-\infty} u(m)$. Since μ_n was arbitrary, the desired limit follows. □

C Proofs for Sections 4.3 and 4.4

Proof of Lemma 3. By Theorem 1 of Jewitt (1989), for any $u_1, u_2 \in U$ and any strategy $\beta \in B^X$,

$$\int u_1 \,\mathrm{d}G_\beta^\mu > \int u_1 \,\mathrm{d}F \quad \text{implies} \quad \int u_2 \,\mathrm{d}G_\beta^\mu \ge \int u_2 \,\mathrm{d}F \tag{4}$$

if either of the following hold: (i) u_1 is more risk-averse than u_2 and $F - G_\beta^\mu$ is single crossing, or (ii) u_2 is more risk-averse than u_1 and $G_\beta^\mu - F$ is single crossing.

Now, to prove the first part, let $u_1, u_2 \in U$ satisfy (ZAZI), with u_1 more risk averse than u_2. If $M(u_1) = 0$, then $M(u_1) \le M(u_2)$. Otherwise, fix $0 \le \mu < M(u_1)$ and let β be optimal for u_1 at price μ with $F - G_\beta^\mu$ single crossing. Since $\mu < M(u_1)$, u_1 strictly prefers purchasing information by Proposition 1, so $\int u_1 \,\mathrm{d}G_\beta^\mu > \int u_1 \,\mathrm{d}F$. By (4), $\int u_2 \,\mathrm{d}G_\beta^\mu \ge \int u_2 \,\mathrm{d}F$, and therefore u_2 weakly prefers purchasing information at price μ. Thus $\mu \le M(u_2)$ by Proposition 1. Since this holds for every $\mu < M(u_1)$, $M(u_1) \le M(u_2)$.

The second part follows by the same argument, taking u_2 to be more (instead of less) risk averse than u_1, and choosing β such that $G_\beta^\mu - F$ is single crossing (instead of $F - G_\beta^\mu$). □

Proof of Lemma 4. Fix $u \in U$ and $\mu \ge 0$, and set

$$X = \{x_1 < \cdots < x_K\}$$

for some $K \in \mathbb{N}$. By Lemma 8, u admits an optimal strategy β and $\int u \,\mathrm{d}G_\beta^\mu \in \mathbb{R} \cup \{-\infty\}$. The result is trivial if either $K = 1$ or $\int u \,\mathrm{d}G_\beta^\mu = -\infty$, hence suppose that $K \ge 2$ and $\int u \,\mathrm{d}G_\beta^\mu$ is finite.

Choose $y_0 < \cdots < y_K$ in $\mathbb{R}$ and set $Y = (y_0, y_K]$. For each $w \in \mathbb{R}$, let $P(\cdot, w)$ be the CDF with support Y and density

$$p_w(y) = \frac{h_w(x_k)}{y_k - y_{k-1}} \qquad \text{for } y \in (y_{k-1}, y_k].$$

Since the experiment α is TP2, the map $(w, y) \mapsto p_w(y)$ is TP2.

Let

$$\{b_1, \ldots, b_n\} = \{\beta_x : x \in X\}, \qquad b_1 \preceq \cdots \preceq b_n,$$

and define $L_i : \mathbb{R} \to \mathbb{R}$ by

$$L_i(w) = -u(w + b_i(w) - \mu)$$

for each $i \in \{1, \ldots, n\}$. Since u is increasing and $b_{i+1} - b_i$ is single crossing, $L_i - L_{i+1}$ is single crossing for each $i < n$.

View β as the decision procedure $Y \to \{1, \ldots, n\}$ that chooses, for each $k \in \{1, \ldots, K\}$ and $y \in (y_{k-1}, y_k]$, $i \in \{1, \ldots, n\}$ such that $\beta_{x_k} = b_i$. Since $P(\cdot|w)$ admits a density for all $w \in \mathbb{R}$, Theorem 1 of Karlin and Rubin (1956) delivers a (deterministic) nondecreasing procedure $\phi : Y \to \{1, \ldots, n\}$ whose risk is weakly lower at every $w \in \mathbb{R}$, i.e.,

$$\sum_{k=1}^{K} \frac{h_w(x_k)}{y_k - y_{k-1}} \int_{y_{k-1}}^{y_k} u(w + b_{\phi(y)}(w) - \mu)\,\mathrm{d}y \geq \sum_{k=1}^{K} h_w(x_k) u(w + \beta_{x_k}(w) - \mu).$$

Integrating with respect to F gives

$$\begin{aligned}\sum_{k=1}^{K} \frac{h(x_k)}{y_k - y_{k-1}} &\int_{y_{k-1}}^{y_k} \int u(w + b_{\phi(y)}(w) - \mu)\,\mathrm{d}F_{x_k}(w)\,\mathrm{d}y \\ &\geq \sum_{k=1}^{K} h(x_k) \int u(w + \beta_{x_k}(w) - \mu)\,\mathrm{d}F_{x_k}(w). \end{aligned} \tag{5}$$

For every $k \in \{1, \ldots, K\}$ and $y \in (y_{k-1}, y_k]$,

$$\int u(w + b_{\phi(y)}(w) - \mu)\,\mathrm{d}F_{x_k}(w) \leq \int u(w + \beta_{x_k}(w) - \mu)\,\mathrm{d}F_{x_k}(w), \tag{6}$$

because β is optimal for u, $h(x_k) > 0$ and $\int u\,\mathrm{d}G^{\mu}_{\beta}$ is finite. Then (5) implies that, for each $k \in \{1, \ldots, K\}$, (6) holds with equality for a.e. $y \in (y_{k-1}, y_k]$, since $h(x_k) > 0$. Consequently, we may choose $z_k \in (y_{k-1}, y_k]$ for each $k \in \{1, \ldots, K\}$ such that the strategy $\beta' \in B^X$ given by $\beta'_{x_k} = b_{\phi(z_k)}$ satisfies $\int u\,\mathrm{d}G^{\mu}_{\beta'} = \int u\,\mathrm{d}G^{\mu}_{\beta}$. Thus, β' is optimal for u, and it is monotone since ϕ is nondecreasing. $\square$

The proof of Lemma 5 relies on Lemma 6 from Section 4.4. We begin by stating the classical result from Karlin and Rubin (1956), then use it to prove Lemma 6, and finally we prove Lemma 5.

Lemma 9 (Karlin and Rubin, 1956)**.** *Let $\phi : \mathbb{R} \to \mathbb{R}$ be single crossing and let $\kappa : \mathbb{R}^2 \to \mathbb{R}_+$ be TP2. Then*

$$m \longmapsto \int \phi(\ell)\kappa(\ell, m)\,\mathrm{d}\ell$$

is single crossing, provided $\ell \mapsto \phi(\ell)\kappa(\ell, m)$ is integrable for every $m \in \mathbb{R}$.

Lemma 6 yields Lemma 5(I) by taking the aggregation kernel from a log-concave density of background wealth. This provides the aggregation step in the proof of Theorem 1. To obtain Lemma 5(II), which involves downward-log-convex background risk, Lemma 6 is applied only to a bounded-above interval of wealth levels, after extending the density to obtain a TP2 kernel; the remaining region is handled by a direct distributional comparison. Thus Theorems 1 and 2 share the same single-crossing principle but require different aggregation arguments.

Proof of Lemma 6. Define

$$\Phi(\ell, m) := \int_{\mathbb{R}} \phi(k, \ell)\kappa(k, m)\,\mathrm{d}k, \qquad \ell, m \in I.$$

Fix $\ell, m \in I$ with $\ell \leq m$ and $\Phi(\ell, \ell) > 0$. By Lemma 9, the map $m' \mapsto \Phi(\ell, m')$ is single crossing, so $\Phi(\ell, m) \geq 0$. Moreover, $\ell > \underline{m}$, since otherwise $\phi(k, \ell) \leq 0$ for every k, contradicting $\Phi(\ell, \ell) > 0$. If $m < \bar{m}$, then $\underline{m} < \ell \leq m < \bar{m}$, and monotonicity in the second argument of ϕ gives

$$\Phi(m, m) \geq \Phi(\ell, m) \geq 0.$$

If $m \geq \bar{m}$, then $\phi(k, m) \geq 0$ for every k, so again $\Phi(m, m) \geq 0$. Thus $\Phi(\ell, \ell) > 0$ and $\ell \leq m$ imply $\Phi(m, m) \geq 0$, proving that $m \mapsto \Phi(m, m)$ is single crossing. □

Proof of Lemma 5. We establish each of the two parts by relying on Lemma 6. For (I), let f_W denote the density of W, and define

$$\phi(\ell, m) = F_{Y^1|W}(\ell|m - \ell) - F_{Y^2|W}(\ell|m - \ell)$$

and

$$\kappa(\ell, m) = f_W(m - \ell).$$

The integrability condition in Lemma 6 holds because $|\phi| \leq 1$. Moreover, for $i \in \{1, 2\}$,

$$\begin{aligned} F_{Y^i+W}(m) &= \int F_{Y^i|W}(m - w|w) f_W(w)\,\mathrm{d}w \\ &= \int F_{Y^i|W}(\ell|m - \ell) f_W(m - \ell)\,d\ell, \end{aligned}$$

where the second equality follows from the change of variable $\ell = m - w$. Hence

$$F_{Y^1+W}(m) - F_{Y^2+W}(m) = \int \phi(\ell, m)\kappa(\ell, m)\,d\ell.$$

For every m, the map $\phi(\cdot, m)$ is single crossing by hypothesis. For every ℓ, the map $\phi(\ell, \cdot)$ is nonpositive on $\{w \in \mathbb{R} : w \leq \underline{w}\}$, nondecreasing on $(\underline{w}, \bar{w})$, and nonnegative on $\{w \in \mathbb{R} : w \geq \bar{w}\}$ by the common hypothesis of the lemma. Since f_W is log concave, κ is

TP2. Therefore, Lemma 6 implies that

$$F_{Y^1+W} - F_{Y^2+W}$$

is single crossing.

It remains to prove (II). Clearly, it suffices to show that $F_{Y^1+W} - F_{Y^2+W}$ is nonnegative on $\{w \in \mathbb{R} : w \geq \bar{w}\}$ and single crossing on $I = (-\infty, \bar{w})$. To this end, let once again f_W denote the density of W, and define $\phi' : \mathbb{R}^2 \to \mathbb{R}$ by

$$\phi'(\ell, m) = F_{Y^1|W}(-\ell|\ell + m) - F_{Y^2|W}(-\ell|\ell + m).$$

For $i \in \{1, 2\}$,

$$\begin{aligned} F_{Y^i+W}(m) &= \int F_{Y^i|W}(m - w|w) f_W(w)\,\mathrm{d}w \\ &= \int F_{Y^i|W}(-\ell|\ell + m) f_W(\ell + m)\,d\ell, \end{aligned}$$

where the second equality follows from the change of variable $\ell = w - m$, so that

$$F_{Y^1+W}(m) - F_{Y^2+W}(m) = \int \phi'(\ell, m) f_W(\ell + m)\,d\ell.$$

For every $m \geq \bar{w}$, the common hypothesis of the lemma implies that $\phi'(\ell, m) \geq 0$ for every ℓ. Hence $F_{Y^1+W}(m) - F_{Y^2+W}(m) \geq 0$ for every $m \geq \bar{w}$.

It remains to show that $D := F_{Y^1+W} - F_{Y^2+W}$ is single crossing on $I := (-\infty, \bar{w})$. Let $q = \log f_W$. Choose $r_n \uparrow \omega$ and a finite subgradient $a_n \in \partial q(r_n)$, and define the convex function

$$q_n(t) = \begin{cases} q(t), & t \leq r_n, \\ q(r_n) + a_n(t - r_n), & t > r_n. \end{cases}$$

Set $c_n := \int_{-\infty}^{\omega} e^{q_n(t)}\,\mathrm{d}t$ and $f_W^n(t) := c_n^{-1} e^{q_n(t)} \mathbf{1}_{\{t \leq \omega\}}$. Since the affine continuation is a supporting line to q, one has $q_n \leq q$ on $(-\infty, \omega)$. Moreover, $q_n(t) = q(t)$ eventually for every fixed $t < \omega$. Dominated convergence therefore gives $c_n \to 1$ and $\|f_W^n - f_W\|_1 \to 0$.

Extend $\log f_W^n$ beyond ω by setting $\widetilde{q}_n(t) := q_n(t) - \log c_n$ for every $t \in \mathbb{R}$, and let $\kappa_n(\ell, m) := e^{\widetilde{q}_n(m+\ell)}$. Convexity of $\widetilde{q}_n$ implies that κ_n is TP2. The support condition gives $\phi'(\ell, m) = 0$ whenever $m \in I$ and $m + \ell > \omega$. Consequently, for every $m \in I$,

$$\Phi_n(m) := \int \phi'(\ell, m)\kappa_n(\ell, m)\,d\ell = \int \phi'(\ell, m) f_W^n(m + \ell)\,d\ell.$$

The assumed single-crossing property of $y \mapsto -\phi'(-y, m)$ implies that $\ell \mapsto \phi'(\ell, m)$ is single crossing, while the common hypotheses give the required monotonicity in m. The integrability condition follows from $|\phi'| \leq 1$ and the support property. Hence Lemma 6 implies that every Φ_n is single crossing on I.

Finally, $|\phi'| \le 1$ gives $\sup_{m\in I} |\Phi_n(m) - D(m)| \le \|f_W^n - f_W\|_1 \to 0$. If $m \le m'$ and $D(m) > 0$, then $\Phi_n(m) > 0$ for all sufficiently large n, so $\Phi_n(m') \ge 0$ and therefore $D(m') \ge 0$. Thus D is single crossing on I. Since D is nonnegative on $[\bar{w}, \infty)$, it is single crossing on $\mathbb{R}$. $\square$

D Worked Investment and Insurance Illustrations

The following pair of examples makes the preceding distributional argument explicit. In both cases the signal reveals whether background wealth lies above or below the common payoff-crossing point of the asset menu. This makes the optimal signal-contingent strategy transparent and allows the induced wealth distribution G^{μ}_{β} to be computed in closed form. We recall the one-parameter family

$$u_q(m) = m - q(-m)^+, \qquad q \ge 0. \tag{7}$$

Each u_q is increasing, concave, and Lipschitz. Moreover, if $q' > q$, then $u_{q'}$ is more risk averse than u_q in the sense of Section 3. The parameter q therefore orders this family by risk aversion; below we evaluate each crossing at the corresponding willingness-to-pay price.

Investment. Let $W \sim N(0,1)$ and set $w_I^* = 1/2$. Consider the investment menu

$$B_I = \{\lambda b_I : \lambda \in [0,1]\}, \qquad b_I(w) = w - w_I^*.$$

The binary signal reveals whether $W < w_I^*$ or $W \ge w_I^*$. This is an MLR-ordered experiment. After the high signal, $b_I(W) \ge 0$ throughout the posterior support, whereas after the low signal, $b_I(W) \le 0$. Hence, for every increasing utility and every information price μ, an optimal strategy is

$$\lambda_L^* = 0, \qquad \lambda_H^* = 1.$$

Thus higher signals induce weakly larger investment positions, as in Lemma 4. The status quo is also optimal before information for every $q \ge 0$: the prior objective is concave in λ, and its right derivative at $\lambda = 0$ is

$$-\frac{1}{2} - q\left(\frac{1}{\sqrt{2\pi}} + \frac{1}{4}\right) < 0.$$

The example therefore satisfies ZAZI for the entire family (7).

After paying μ, terminal wealth is

$$T_I(W) = \begin{cases} W - \mu, & W < w_I^*, \\ 2W - w_I^* - \mu, & W \geq w_I^*. \end{cases}$$

Since this map is increasing and continuous, its distribution can be obtained by inversion. Writing $F = \Phi$ for the standard-normal CDF,

$$G_I^\mu(m) = \begin{cases} F(m + \mu), & m < w_I^* - \mu, \\ F\Big(\dfrac{m + \mu + w_I^*}{2}\Big), & m \geq w_I^* - \mu. \end{cases} \tag{8}$$

For $m < w_I^* - \mu$, $F(m) - G_I^\mu(m) \leq 0$, with strict inequality if $\mu > 0$. For $m \geq w_I^* - \mu$, its sign is the sign of

$$m - \frac{m + \mu + w_I^*}{2}.$$

Thus $F - G_I^\mu$ is single crossing. If $\mu > 0$, its unique strict sign change occurs at

$$m_I^{SC} = w_I^* + \mu, \tag{9}$$

from negative to positive. If $\mu = 0$, the difference is zero below w_I^* and nonnegative above. This is the orientation required in the first part of Lemma 3.

Insurance. For a mirror-image example satisfying the support assumption of Theorem 2, let background wealth have the reflected-Lomax distribution

$$F(w) = \frac{1}{(2 - w)^3}, \qquad w \leq 1,$$

with $F(w) = 1$ for $w > 1$. Its density is $f(w) = 3(2 - w)^{-4}$ on $(-\infty, 1]$, so $\log f$ is strictly convex and the support is unbounded below. Set $w_P^* = -1/2$, $a = 3/5$, and consider

$$B_P = \{\lambda b_P : \lambda \in [0, 1]\}, \qquad b_P(w) = a(w_P^* - w).$$

Because $a \in (0, 1)$, b_P is nonincreasing and $w + b_P(w)$ is increasing, so this is an insurance menu. Let the signal reveal whether $W \leq w_P^*$ or $W > w_P^*$. After the low signal the insurance payoff is nonnegative throughout the posterior support, while after the high signal it is nonpositive. Thus an optimal strategy is

$$\lambda_L^* = 1, \qquad \lambda_H^* = 0,$$

so higher signals induce weakly smaller insurance positions. For the family (7), the prior objective is concave in λ, and its right derivative at zero is

$$-\frac{3}{5} + \frac{3q}{80}.$$

Hence ZAZI holds for every $q \leq 16$, in particular for all the values reported below.

Terminal wealth after purchasing information at price μ is

$$T_P(W) = \begin{cases} (1-a)W + aw_P^* - \mu, & W \leq w_P^*, \\ W - \mu, & W > w_P^*. \end{cases}$$

Again the map is increasing and continuous, and therefore

$$G_P^\mu(m) = \begin{cases} F\left(\dfrac{m + \mu - aw_P^*}{1-a}\right), & m < w_P^* - \mu, \\ F(m+\mu), & m \geq w_P^* - \mu. \end{cases} \tag{10}$$

In the lower region, the sign of $G_P^\mu(m) - F(m)$ is the sign of

$$\frac{m + \mu - aw_P^*}{1-a} - m,$$

which vanishes at

$$m_P^{SC} = w_P^* - \frac{\mu}{a}. \tag{11}$$

Above $w_P^* - \mu$, $G_P^\mu(m) - F(m) = F(m+\mu) - F(m) \geq 0$. Thus $G_P^\mu - F$ is single crossing. If $\mu > 0$, its unique strict sign change is from negative to positive at m_P^{SC}; if $\mu = 0$, it is negative below w_P^* and zero above. This is the orientation required in the second part of Lemma 3.

Let $M_I(u_q)$ and $M_P(u_q)$ denote willingness to pay in the investment and insurance examples. The following table reports these values and the associated crossing points. The scalar indifference equations are easily solved numerically.

q	$M_I(u_q)$	m_I^{SC}	$M_P(u_q)$	m_P^{SC}
0.5	0.1563	0.6563	0.0676	-0.6126
1	0.1296	0.6296	0.0847	-0.6412
2	0.0970	0.5970	0.1131	-0.6885
4	0.0648	0.5648	0.1534	-0.7557

Thus willingness to pay falls with risk aversion in the investment example and rises with risk aversion in the insurance example. The optimal strategy is independent of q; across rows, q affects the induced distribution only through the price $\mu = M(u_q)$.

These examples are intentionally stronger than what the main theorems require. Because the signal reveals which side of the common payoff-crossing point has occurred, the optimal action is pointwise dominant within each posterior support and the distribution G^{μ}_{β} can be inverted directly. The examples therefore illustrate the first two ingredients of the proof, the monotone strategy and the resulting single-crossing wealth comparison, but they do not by themselves explain why log concavity or downward log convexity is needed.

With general MLR signals, posterior supports may overlap, and the conditional comparisons must then be aggregated across background-wealth realizations. The tail assumptions enter precisely in preserving the relevant crossing through that aggregation step.

E Additional Lemmas for Section 4.5

The following auxiliary lemmas establish the technical ingredients used in the proofs of the main theorems. The first derives first-order stochastic dominance from TP2, while the latter two analyze the conditional wealth distributions induced by monotone strategies.

Lemma 10 (TP2 and stochastic monotonicity)**.** *Let $X = \{x_1 < \cdots < x_K\}$, and suppose that the kernel $(w, x) \mapsto h_w(x)$ is TP2. Then, whenever $w \leq w'$, the distribution $h_{w'}$ first-order stochastically dominates h_w.*

Proof. Fix $w \leq w'$ and set $a_i = h_w(x_i)$ and $b_i = h_{w'}(x_i)$. TP2 gives $a_i b_j \geq a_j b_i$ for $i < j$. Hence $b_i > a_i$ implies $b_j \geq a_j$ for every $j > i$, so $(b_i - a_i)_{i=1}^K$ changes sign at most once, from nonpositive to nonnegative. Since $\sum_{i=1}^K (b_i - a_i) = 0$, it follows that $\sum_{i=1}^k b_i \leq \sum_{i=1}^k a_i$ for every $k < K$, which is the claimed first-order stochastic dominance. □

Lemma 11. *Let B be an investment menu with threshold $w^* \in \mathbb{R}$ and complete ordering $\succeq$, and let $\beta \in B^X$ be monotone relative to $\succeq$. Let χ denote the signal generated by the experiment, and define*

$$Y^1 = 0, \qquad Y^2 = \beta_\chi(W) - \mu.$$

Then versions of $F_{Y^1|W}$ and $F_{Y^2|W}$ can be chosen so as to satisfy the hypotheses of Lemma 5(I).

Proof of Lemma 11. Choose versions such that

$$\begin{aligned} F_{Y^1|W}(y|z) &= \mathbf{1}_{\mathbb{R}_+}(y), \\ F_{Y^2|W}(y|z) &= \sum_{x \in X} h_z(x) \mathbf{1}_{\{\beta_x(z) - \mu \leq y\}}, \end{aligned}$$

and write

$$\Delta(y, w) = \mathbf{1}_{\mathbb{R}_+}(y) - \sum_{x \in X} h_{w-y}(x) \mathbf{1}_{\{\beta_x(w-y) - \mu \leq y\}}.$$

For every w, $\Delta(y,w) \le 0$ when $y < 0$ and $\Delta(y,w) \ge 0$ when $y \ge 0$, so $y \mapsto \Delta(y,w)$ is single crossing.

Every nonzero asset is ranked above $\mathbf{0}$: if $\mathbf{0} \succeq b$, then $b \ge 0$ below w^* and $b \le 0$ above w^*, which together with monotonicity of b implies $b = \mathbf{0}$. Hence $b(z) \le 0$ for every $b \in B$ and $z \le w^*$. Fix y. If $w \le w^*$ and $y < 0$, then $\Delta(y,w) \le 0$; if $y \ge 0$, then $w - y \le w^*$ and every indicator equals one, so $\Delta(y,w) = 0$. Thus $\Delta(y,\cdot)$ is nonpositive on $(-\infty, w^*]$.

Now fix $w^* \le w < w'$, set $z = w - y$ and $z' = w' - y$, and let $I_x(t) = \mathbf{1}_{\{\beta_x(t)-\mu \le y\}}$. Since every asset is nondecreasing, $I_x(z) \ge I_x(z')$. Moreover, $x \mapsto I_x(z')$ is nonincreasing. Indeed, if $z' > w^*$, this follows from monotonicity of β and the investment-menu ordering; if $z' \le w^*$, then $y = w' - z' > 0$ and every indicator equals one. Since $z \le z'$, Lemma 10 yields

$$\begin{aligned} F_{Y^2|W}(y|z) &= \sum_{x \in X} h_z(x) I_x(z) \\ &\ge \sum_{x \in X} h_z(x) I_x(z') \\ &\ge \sum_{x \in X} h_{z'}(x) I_x(z') = F_{Y^2|W}(y|z'). \end{aligned}$$

Therefore $\Delta(y,w') \ge \Delta(y,w)$. The hypotheses of Lemma 5(I) hold with $\underline{w} = w^*$ and $\bar{w} = \infty$. $\square$

Lemma 12. *Let random wealth W have support $(-\infty, \omega]$, let B be an insurance menu with threshold $w^* \in (-\infty, \omega)$ and witnessing complete ordering $\succeq$, and let $\beta \in B^X$ be monotone relative to the reverse of $\succeq$. Let χ denote the signal generated by the experiment, and define*

$$Y^1 = \beta_\chi(W) - \mu, \qquad Y^2 = 0.$$

Then versions of $F_{Y^1|W}$ and $F_{Y^2|W}$ can be chosen so as to satisfy the hypotheses of Lemma 5(II).

Proof of Lemma 12. Choose versions such that

$$\begin{aligned} F_{Y^1|W}(y|z) &= \sum_{x \in X} h_z(x) \mathbf{1}_{\{\beta_x(z)-\mu \le y\}}, \\ F_{Y^2|W}(y|z) &= \mathbf{1}_{\mathbb{R}_+}(y), \end{aligned}$$

and write

$$\Delta(y,w) = \sum_{x \in X} h_{w-y}(x) \mathbf{1}_{\{\beta_x(w-y)-\mu \le y\}} - \mathbf{1}_{\mathbb{R}_+}(y).$$

For every w, $-\Delta(y,w) \le 0$ when $y < 0$ and $-\Delta(y,w) \ge 0$ when $y \ge 0$, so $y \mapsto -\Delta(y,w)$ is single crossing.

Every nonzero asset is ranked above $\mathbf{0}$: if $\mathbf{0} \succeq b$, then $b \leq 0$ below w^* and $b \geq 0$ above w^*, which together with monotonicity of b implies $b = \mathbf{0}$. Hence every $b \in B$ is nonnegative below w^*, nonpositive above w^*, and vanishes at w^*. Moreover, every insurance payoff is continuous: if $s < t$, then $0 \leq b(s) - b(t) \leq t - s$ because b is nonincreasing and $w \mapsto w + b(w)$ is nondecreasing.

Define

$$\bar{w} = \inf\{w \in \mathbb{R} : b(w) \leq \mu \text{ for every } b \in B\}.$$

The set is a closed upper interval. For every $b \in B$,

$$w^* - \mu + b(w^* - \mu) \leq w^* + b(w^*) = w^*,$$

so $b(w^* - \mu) \leq \mu$ and $\bar{w} \leq w^* - \mu$.

Fix y. If $w \geq \bar{w}$ and $y < 0$, then $\Delta(y, w) \geq 0$. If $y \geq 0$, set $z = w - y$. Since $w \mapsto w + \beta_x(w)$ is nondecreasing and $\beta_x(w) \leq \mu$,

$$z + \beta_x(z) \leq w + \beta_x(w),$$

hence $\beta_x(z) - \mu \leq y$ for every x. Thus all indicators equal one and $\Delta(y, w) = 0$.

Now fix $w < w' < \bar{w}$, set $z = w - y$ and $z' = w' - y$, and let $I_x(t) = \mathbf{1}_{\{\beta_x(t) - \mu \leq y\}}$. Since every insurance payoff is nonincreasing, $I_x(z) \leq I_x(z')$. We claim that $x \mapsto I_x(z')$ is nondecreasing. If $z' < w^*$, this follows from monotonicity of β relative to the reverse menu order. If $z' \geq w^*$, then $y = w' - z' \leq 0$. Since $w' < \bar{w}$, choose $b_0 \in B$ with $b_0(w') > \mu$. For each x, completeness allows us to choose $b \succeq \beta_x$ with $b(w') > \mu$: take $b = b_0$ if $b_0 \succeq \beta_x$ and $b = \beta_x$ otherwise. Since $z' \geq w^*$ and $w' < w^*$,

$$\beta_x(z') \geq b(z') \quad \text{and} \quad b(z') - y = z' + b(z') - w' \geq b(w') > \mu.$$

Hence $\beta_x(z') - \mu > y$, so every indicator equals zero.

Since $z \leq z'$, Lemma 10 gives

$$\begin{aligned} F_{Y^1|W}(y|z) &= \sum_{x \in X} h_z(x) I_x(z) \\ &\leq \sum_{x \in X} h_z(x) I_x(z') \\ &\leq \sum_{x \in X} h_{z'}(x) I_x(z') = F_{Y^1|W}(y|z'). \end{aligned}$$

Thus $\Delta(y, w') \geq \Delta(y, w)$.

Finally, fix $w < \bar{w}$ and $y < w - \omega$, and set $\ell = w - y > \omega > w^*$. Since every asset vanishes at w^* and total wealth is nondecreasing,

$$\ell + \beta_x(\ell) - \mu \geq w^* + \beta_x(w^*) - \mu = w^* - \mu \geq \bar{w} > w.$$

Thus $\beta_x(\ell) - \mu > y$ for every x, while $y < 0$. Both conditional distribution functions therefore vanish at (y, ℓ), so $\Delta(y, w) = 0$. All the hypotheses of Lemma 5(II) hold with $\underline{w} = -\infty$. $\square$

F Proofs of Section 5

The two reversal proofs use different tail comparisons and calibrations. The investment proof uses adjacent-mass geometry and calibrates a one-parameter family of menus by the intermediate value theorem. The insurance proof uses the extreme-tail implication of super-exponentiality and sets the information price equal to the expected gross insurance payoff of a risk-neutral benchmark. A local increase in risk aversion then produces each reversal.

F.1 Ingredients of the Investment Reversal

For adjacent intervals of lengths a and μ, $P_{a,\mu}(m)$ is the share of the mass between $m - a$ and $m + \mu$ that lies above m. The next lemma shows that this share increases as the intervals move right under strict log convexity and decreases under strict log concavity; both directions are also used in Section 6.1.

Lemma 13 (Left-tail geometry)**.** *Suppose that f is continuously differentiable and strictly positive on $(-\infty, \omega)$. Fix $a, \mu > 0$, and define*

$$P_{a,\mu}(m) = \frac{F(m+\mu) - F(m)}{F(m+\mu) - F(m-a)}$$

whenever $m + \mu < \omega$.

(i) If $\log f$ is strictly convex on $(-\infty, \omega)$, then $P_{a,\mu}$ is strictly increasing.

(ii) If $\log f$ is strictly concave on $(-\infty, \omega)$, then $P_{a,\mu}$ is strictly decreasing.

Proof. Set $A^+(m) = F(m+\mu) - F(m)$ and $A^-(m) = F(m) - F(m-a)$. Since $P_{a,\mu} = A^+/(A^+ + A^-)$, it has the same monotonicity as A^+/A^-. Writing $r = (\log f)' = f'/f$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}m} \log \frac{A^+(m)}{A^-(m)} = \frac{\int_m^{m+\mu} r(t) f(t)\,\mathrm{d}t}{\int_m^{m+\mu} f(t)\,\mathrm{d}t} - \frac{\int_{m-a}^{m} r(t) f(t)\,\mathrm{d}t}{\int_{m-a}^{m} f(t)\,\mathrm{d}t}.$$

The two terms are the $f(t)\,\mathrm{d}t$-weighted averages of r over the adjacent intervals $[m, m+\mu]$ and $[m-a, m]$. If $\log f$ is strictly convex, then r is nondecreasing and nonconstant on

$[m-a, m+\mu]$, so the first average is strictly larger than the second. Thus A^+/A^-, and hence $P_{a,\mu}$, is strictly increasing. Under strict log concavity the inequalities are reversed, so $P_{a,\mu}$ is strictly decreasing. $\square$

F.2 Proof of Proposition 2

The proof proceeds in three steps. First, we construct a one-parameter family of binary investment menus under a fixed extreme TP2 experiment, with the same strategy strictly optimal throughout the family and the status quo strictly optimal under the prior. Second, we calibrate the parameters so that the value of information at the prescribed price varies continuously across the family and crosses zero, allowing the asset to be selected by the intermediate value theorem. Third, Lemma 13 determines the sign of the distributional comparison generated by the calibrated asset, and a local perturbation of the benchmark utility completes the proof.

Construction of the calibration family. Since f is integrable on $(-\infty, \omega)$ and $\log f$ is strictly convex, there exists $w_* < \omega$ at which f is locally strictly increasing. For parameters $\mu > 0$ and $\bar{w}$ satisfying

$$w_* < \bar{w} < \omega - \mu,$$

set

$$q = P_{\mu,\mu}(w_*) = \frac{F(w_* + \mu) - F(w_*)}{F(w_* + \mu) - F(w_* - \mu)}.$$

For $\varepsilon, \delta \in (0,1)$, define

$$v_\varepsilon(w) = \begin{cases} w, & w \le \bar{w}, \\ \bar{w} + \varepsilon(w - \bar{w}), & w > \bar{w}, \end{cases}$$

and

$$u_{\varepsilon,\delta}(w) = \begin{cases} v_\varepsilon(w), & w \le w_*, \\ v_\varepsilon(w_*) + \delta\big(v_\varepsilon(w) - v_\varepsilon(w_*)\big), & w > w_*. \end{cases}$$

Both utilities belong to U, and $u_{\varepsilon,\delta}$ is more risk averse than v_ε.

For $\widehat{w} < w_* - \mu$ and $L > 0$, consider the binary experiment

$$h_w(1) = \begin{cases} 0, & w \le \widehat{w}, \\ q, & w > \widehat{w}, \end{cases} \qquad h_w(0) = 1 - h_w(1).$$

The experiment is TP2. For each $c \in [\mu, 2\mu]$, define

$$b_c(w) = \begin{cases} -L, & w < \widehat{w}, \\ 0, & w = \widehat{w}, \\ c, & w > \widehat{w}. \end{cases}$$

Let $B_c = \{\mathbf{0}, b_c\}$. Since b_c is nondecreasing, nonpositive below $\widehat{w}$, nonnegative above $\widehat{w}$, and vanishes at $\widehat{w}$, B_c is an investment menu with threshold $\widehat{w}$. Consider the strategy

$$\beta_0^c = \mathbf{0}, \qquad \beta_1^c = b_c.$$

Calibration of the parameters. Choose $\mu > 0$ sufficiently small that $w_* + \mu < \omega$ and $q = P_{\mu,\mu}(w_*) > \frac{1}{2}$.

Consider first the auxiliary capped utility $v_0(w) = \min\{w, \bar{w}\}$, where $\bar{w}$ will be selected below. At $c = 2\mu$, first consider the limiting experiment obtained by sending $\widehat{w}$ to $-\infty$. Under the strategy $\beta^{2\mu}$, terminal wealth then equals $W + \mu$ with probability q and $W - \mu$ with probability $1 - q$. Its distribution $\widetilde{G}$ satisfies

$$\widetilde{G}(m) = qF(m - \mu) + (1 - q)F(m + \mu).$$

For every m such that $m + \mu < \omega$, $\widetilde{G}(m) > F(m)$ iff $q < P_{\mu,\mu}(m)$. Since $q = P_{\mu,\mu}(w_*)$ and $P_{\mu,\mu}$ is strictly increasing, $\widetilde{G}(m) < F(m)$ for every $m < w_*$, while $\widetilde{G}(m) > F(m)$ for every $w_* < m < \omega - \mu$. In particular, $\int_{-\infty}^{w_*} \big(F(m) - \widetilde{G}(m)\big)\, \mathrm{d}m > 0$.

Choose $\bar{w} \in (w_*, \omega - \mu)$ sufficiently close to w_* that $\int_{-\infty}^{\bar{w}} \big(F(m) - \widetilde{G}(m)\big)\, \mathrm{d}m > 0$. Integration by parts gives $\int v_0 \, d(\widetilde{G} - F) > 0$.

We next choose the cutoff $\widehat{w}$. Let $G_{2\mu}^{\widehat{w}}$ denote the terminal-wealth distribution induced by the actual experiment and the strategy $\beta^{2\mu}$. The actual and limiting constructions differ only when $W \le \widehat{w}$. Since v_0 is 1-Lipschitz,

$$\left| \int v_0 \, \mathrm{d}G_{2\mu}^{\widehat{w}} - \int v_0 \, d\widetilde{G} \right| \le 2q\mu F(\widehat{w}).$$

Choose $\widehat{w} < w_* - \mu$ sufficiently far to the left that $\int v_0 \, d(G_{2\mu}^{\widehat{w}} - F) > 0$.

Having fixed $\widehat{w}$, choose $L > 0$ sufficiently large that

$$-LF(\widehat{w}) + 2\mu\big(1 - F(\widehat{w})\big) < 0.$$

For v_ε, the utility gain from b_c equals $-L$ on $(-\infty, \widehat{w})$, equals zero at $\widehat{w}$, and is at most 2μ on $(\widehat{w}, \infty)$. Thus the displayed bound makes the status quo strictly optimal under the prior, uniformly over $c \in [\mu, 2\mu]$ and $\varepsilon \in [0, 1]$. The unnormalized gain after signal 0 is bounded above by $-LF(\widehat{w}) + 2\mu(1 - q)(1 - F(\widehat{w}))$, which is smaller and therefore negative. After signal 1, the posterior is supported on $(\widehat{w}, \infty)$, where $b_c = c > 0$, so b_c is strictly optimal. Hence β^c is the unique optimal informed strategy throughout the family.

For $c \in [\mu, 2\mu]$, let G_c denote the terminal-wealth distribution induced by β^c, and define $S_\varepsilon(c) = \int v_\varepsilon \, d(G_c - F)$. At $c = \mu$, purchasing information leaves wealth unchanged after signal 1 and lowers it by μ after signal 0. Hence $S_0(\mu) < 0$. By the choice of $\widehat{w}$, $S_0(2\mu) > 0$. Continuity in c therefore allows us to choose $\mu < c_- < c_+ < 2\mu$ such that $S_0(c_-) < 0 < S_0(c_+)$.

We now choose ε. For every $c \in [c_-, c_+]$ and every $m \in [w_*, \bar{w}]$, one has $m + \mu - c \geq w_* - \mu > \widehat{w}$. Hence $G_c(m) = qF(m+\mu-c) + (1-q)F(m+\mu)$. Consequently, $G_c(m) > F(m)$ iff $q < P_{c-\mu,\mu}(m)$.

Since $c < 2\mu$, one has $c - \mu < \mu$. The map $a \mapsto P_{a,\mu}(m)$ is strictly decreasing, and Lemma 13(i) gives $P_{c-\mu,\mu}(m) > P_{\mu,\mu}(m) \geq P_{\mu,\mu}(w_*) = q$.

Thus $G_c(m) > F(m)$ throughout $[w_*, \bar{w}] \times [c_-, c_+]$. By continuity and compactness,

$$A := \min_{c \in [c_-, c_+]} \int_{w_*}^{\bar{w}} \big(G_c(m) - F(m)\big) \, \mathrm{d}m > 0.$$

Moreover, since the terminal wealth induced by β^c has uniformly bounded first moments over $c \in [c_-, c_+]$,

$$K := \sup_{c \in [c_-, c_+]} \int_{\bar{w}}^{\infty} |G_c(m) - F(m)| \, \mathrm{d}m < \infty.$$

Choose $\varepsilon > 0$ sufficiently small that $\varepsilon K < A$ and $S_\varepsilon(c_-) < 0 < S_\varepsilon(c_+)$.

Since $c \mapsto b_c$ is continuous in $L^1(F)$ and v_ε is Lipschitz continuous, S_ε is continuous on $[c_-, c_+]$. Hence there exists $c^* \in (c_-, c_+)$ such that $S_\varepsilon(c^*) = 0$.

For the menu B_{c^*}, the status quo is strictly optimal under the prior and β^{c^*} is the unique optimal informed strategy. Hence $M(v_\varepsilon) = \mu$.

It remains to choose δ. Write $G = G_{c^*}$ and $v = v_\varepsilon$. Since $u_{\varepsilon,\delta}(w) = v(w) + (\delta - 1)\big(v(w) - v(w_*)\big)^+$, integration by parts gives

$$\begin{aligned} \int \big(v - v(w_*)\big)^+ \, d(G - F) &= -\int_{w_*}^{\bar{w}} \big(G(m) - F(m)\big) \, \mathrm{d}m \\ &\quad - \varepsilon \int_{\bar{w}}^{\infty} \big(G(m) - F(m)\big) \, \mathrm{d}m \\ &\leq -A + \varepsilon K < 0. \end{aligned}$$

Since $\int v \, d(G - F) = 0$, it follows that, for every $\delta < 1$, $\int u_{\varepsilon,\delta} \, d(G - F) > 0$. Finally, choose $\delta < 1$ sufficiently close to 1 that the strict prior optimality of the status quo is preserved, and set $v = v_\varepsilon$ and $u = u_{\varepsilon,\delta}$. Then both utilities satisfy (ZAZI), u is more risk averse than v, and $M(v) = \mu$. The strategy β^{c^*} yields strictly positive information surplus for u at price μ, so Proposition 1 gives $M(u) > \mu = M(v)$. Since B_{c^*} is an investment menu and α is TP2, this proves Proposition 2.

F.3 Proof of Proposition 3

We first note that super-exponentiality implies that $\lim_{m\to-\infty} F(m+\Delta)/F(m) = +\infty$ for every $\Delta > 0$. Indeed, fix $K > 0$. For all sufficiently negative m, $(\log f)'(s) \geq K$ whenever $s \leq m+\Delta$. Hence $f(s+\Delta) \geq e^{K\Delta} f(s)$ for every $s \leq m$. Integrating with respect to s over $(-\infty, m]$ gives $F(m+\Delta) \geq e^{K\Delta} F(m)$. Since K is arbitrary, the claim follows.

We next construct the benchmark decision problem. Fix $q \in (0,1)$ and $a, \ell > 0$. For $z \in \mathbb{R}$, define $b_z(w) = \max\{-\ell, \min\{a, z-w\}\}$. The payoff b_z is nonincreasing, $w + b_z(w)$ is nondecreasing, and b_z is nonnegative below z and nonpositive above z. Thus $B_z = \{\mathbf{0}, b_z\}$ is an insurance menu with threshold z. Moreover, $b_z(w) \to -\ell$ as $z \to -\infty$ for every w, while $|b_z| \leq \max\{a, \ell\}$. Hence, by dominated convergence, $\int b_z \,\mathrm{d}F \to -\ell$. Choose z sufficiently negative that $\int b_z \,\mathrm{d}F < 0$, write $b = b_z$ and $B = B_z$, and choose $\widehat{w} < \min\{z-a, \omega\}$.

Let $X = \{0,1\}$ and define the experiment α by $h_w(0) = q\mathbf{1}_{\{w \leq \widehat{w}\}}$ and $h_w(1) = 1 - h_w(0)$. Since $w \mapsto h_w(0)$ is nonincreasing, this binary experiment is TP2. Set $p = F(\widehat{w})$. Because $\widehat{w} < \omega$ and f is strictly positive on $(-\infty, \omega)$, $p \in (0,1)$. Hence both signals have positive ex ante probability. Let $v(w) = w$, set $\mu = pqa \in (0, a)$, and consider the strategy $\beta_0 = b$ and $\beta_1 = \mathbf{0}$.

Because v is linear, the common information price does not affect the action comparisons. Then, at the prior, the expected net gain of choosing b is $\int b \,\mathrm{d}F < 0$, so that v satisfies (ZAZI). Since the expected net gain after signal 0 is $a > 0$, the expected net gain after signal 1 is strictly negative and, hence, β is optimal for v. Since signal 0 has ex-ante probability pq, it follows that the expected net gain of acquiring information (and acting optimally) is $pqa - \mu = 0$, so that $M(v) = \mu$.

We now identify the extreme-tail deterioration that greater risk aversion will amplify. Let $G = G^{\mu}_{\beta}$. For every $m < \widehat{w} - \mu$,

$$G(m) = qF(m+\mu-a) + (1-q)F(m+\mu), \qquad \frac{G(m)}{F(m)} \geq (1-q)\frac{F(m+\mu)}{F(m)} \longrightarrow +\infty.$$

Thus there exists $t < \widehat{w} - \mu$ such that $G(m) - F(m) > 0$ for every $m < t$. Since F and G have finite first moments, $I := \int_{-\infty}^{t} (G(m) - F(m)) \,\mathrm{d}m$ is finite and strictly positive.

For $\lambda > 1$, define $u_\lambda(w) = w - (\lambda - 1)(t-w)^+$. This utility is increasing, Lipschitz continuous, and concave, hence more risk averse than v. Since F and G have the same mean and $(t-w)^+ = \int_{-\infty}^{t} \mathbf{1}_{\{w \leq m\}} \,\mathrm{d}m$, integration by parts gives

$$\int u_\lambda \,\mathrm{d}(G - F) = -(\lambda - 1)\int_{-\infty}^{t} \big(G(m) - F(m)\big) \,\mathrm{d}m = -(\lambda-1)I < 0.$$

At $\lambda = 1$, the prior action comparison and both posterior action comparisons are strict.

Their expected-utility differences are continuous in λ by dominated convergence, so the same signs hold for some $\lambda > 1$ sufficiently close to one. For $u = u_\lambda$, the status quo remains uniquely optimal under the prior and β remains the unique optimal informed strategy at price μ. Thus u satisfies (ZAZI) but strictly prefers not to purchase information at $\mu = M(v)$, and Proposition 1 yields $M(u) < M(v)$.

G Proofs of Lemmas Used in Examples

Lemma 14 (Two-exponential ratio)**.** *Let $a, b > 0$, $c \geq 0$, and $0 \leq \lambda_0 < \lambda_1$. For $r > 0$, set $X_\lambda(r) = ae^{-cr} + be^{-\lambda r}$ and*

$$j(r) = -\frac{1}{r} \log \frac{X_{\lambda_1}(r)}{X_{\lambda_0}(r)}.$$

If $\lambda_1 \leq c$, then j is strictly increasing; if $c \leq \lambda_0$, then j is strictly decreasing.

Proof. Set $\Phi_\lambda(r) = \log X_\lambda(r) - r\partial_r X_\lambda(r)/X_\lambda(r)$. Direct differentiation gives

$$j'(r) = \frac{\Phi_{\lambda_1}(r) - \Phi_{\lambda_0}(r)}{r^2}, \qquad \partial_\lambda \Phi_\lambda(r) = \frac{abr^2(c-\lambda)e^{-(c+\lambda)r}}{X_\lambda(r)^2}.$$

Thus $\Phi_\lambda(r)$ is strictly increasing in λ below c and strictly decreasing above c. Since $\lambda_0 < \lambda_1$, integrating over $[\lambda_0, \lambda_1]$ gives the two strict conclusions. □

Proof of Lemma 1. Let $q = 1 - p$ and $d = k_H - k_L$. Factoring e^{-rk_L} shows that the ratio inside the logarithm is $X_{d+H}(r)/X_d(r)$ for $X_\lambda(r) = p + qe^{-\lambda r}$. Apply Lemma 14 with $a = p$, $b = q$, $c = 0$, $\lambda_0 = d$, and $\lambda_1 = d + H$. □

Proof of Lemma 2. Let $q = 1 - p$ and $\Delta = k_H - k_L$. Factoring e^{-rk_L} shows that the ratio inside the logarithm is $X_L(r)/X_0(r)$ for $X_\lambda(r) = qe^{-\Delta r} + pe^{-\lambda r}$. Since $0 < L < \Delta$, apply Lemma 14 with $a = q$, $b = p$, $c = \Delta$, $\lambda_0 = 0$, and $\lambda_1 = L$. □

Proof of Lemma 7. Set $\delta = k_H - k_L$ and, for $a, r > 0$, let $\varphi_r(a) = a/(e^{ar} - 1)$. Then

$$\frac{\mathrm{d}}{\mathrm{d}r} \log \rho(r) = -\delta + \varphi_r(H_E) - \varphi_r(L_P).$$

The inequalities $1 - e^{-ar} < ar < e^{ar} - 1$ imply

$$\frac{e^{-ar}}{r} < \varphi_r(a) < \frac{1}{r}.$$

Hence

$$\varphi_r(H_E) - \varphi_r(L_P) < \frac{1 - e^{-L_P r}}{r} < L_P,$$

so $(\log \rho)'(r) < -\delta + L_P < 0$. Since $\rho > 0$, it is strictly decreasing. Moreover,

$$\lim_{r \downarrow 0} \rho(r) = \frac{1-p}{p} \frac{H_E}{L_P}, \qquad \lim_{r \to \infty} \rho(r) = 0.$$

If the first limit is at most one, strict decrease gives $\rho(r) < 1$ for every $r > 0$. If it exceeds one, continuity and strict decrease give a unique $r_0 > 0$ such that $\rho(r_0) = 1$, with $\rho(r) > 1$ for $r < r_0$ and $\rho(r) < 1$ for $r > r_0$. $\square$